Appendix to Alpha Duties: The Search for Excess Returns and Appropriate Fiduciary Duties

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Data

Unless otherwise indicated, the data for this study are drawn from the Center for Research in Security Prices. We use monthly data on all firms in the CRSP which includes all securities traded on the NYSE (from 1926), AMEX (from 1962), and NASDAQ (from 1972) and NYSE ARCA (from 2006) to December 2016.[1] Data on the Consumer Price Index for All Urban Consumers (CPI) used to create real returns, because it is available monthly back to 1925.[2]

Costs of Failure to Diversify

Baseline Result

Our baseline attempt to quantify the costs of failing to diversify (among equities) looks at investors with preferences which exhibit constant relative risk aversion, a one year holding period, and no outside income. We then ask, how much alpha would one have to add to make the investor indifferent between holding a single publicly traded stock and a diversified portfolio of stocks (i.e., the market basket of U.S. equities)?

To answer this question, we intuitively need to know how much riskier a single stock is than the market. Our estimates of this additional risk is based on Monte Carlo simulation, which uses monthly historical stock data from 1925 to 2016 to build a distribution of probable returns for both a diversified portfolio and individual stocks.[3] We split the sample into "normal" periods, and "crisis" periods, by looking at the volatility of the market over the prior 30 (calendar) days. If the annualized market volatility over the past 30 days was over 25%, then we classify the month as a crisis period.[4]

To construct the individual stock distribution during normal periods, we randomly draw (with replacement) 100,000 individual stocks from across the normal months—where the probability of selecting a stock is proportional to its share of the total market capitalization

[1] In addition to common equity shares for firms incorporated in the US, this includes:

(a) common shares of closed end funds, real estate investment trusts, and companies incorporated outside the United States

(b) ADR's and shares of beneficial interest.

Excluding the types of securities listed in (a) and (b) has very little effect on the results.

[2] https://fred.stlouisfed.org/series/CPIAUCNS

[3] Put differently, we look to historical data to estimate the probability that a individual stock or a diversified portfolio whole will rise by 10% or fall by 10% in a year, rise by 11% or fall by 11% etc.

[4] While we believe it is useful to use a Monte-Carlo simulation in this context to build the distributions of individual stocks and the market, a roughly similar result can be reached by looking to historical data to find the average standard deviation and mean of the annual returns on individual stocks and the full market, and then assuming both will simply follow a normal distribution.

during that month—and calculate the *real* return of that stock for the following year. We do the same for crisis months.[5]

Likewise, we follow a similar procedure for the diversified portfolio, except that there is an equal probability of selecting any observation in the sample. We calculate the alpha as the amount we need to increase the return of each firm return before the investor is indifferent between facing the distribution of firm returns as the distribution of market returns.

The results are produced in Table 1, reproduced below:

Table A1.1
By How Much Must a Risk-Averse Person Outperform the Market to Make Holding a Single Stock Worthwhile

	Additional Alpha Required
CRRA Coefficient = 1 (Log Utility)	
Regular Period	3.1%
Crisis Period	5.0%
CRRA Coefficient = 2	
Regular Period	6.5%
Crisis Period	9.8%
CRRA Coefficient = 3	
Regular Period	10.4%
Crisis Period	14.7%
CRRA Coefficient = 4	
Regular Period	15.0%
Crisis Period	19.0%

Notes: The holding period is one year. The distribution of both the market and single-stock returns is constructed by randomly sampling from the historical returns of firms in the CRSP database from 1926-2015 (i.e., by Monte Carlo simulation). For the market, we use the market-cap-weighted average of all of the CRSP firms. We then calculate the real 1-year return for each month for this market portfolio during 1926-2015, and then randomly select (with replacement) 100,000 of the 1 year returns to create a distribution. Similarly, we calculate a 1-year real return for every firm for each month during the period and then randomly select 100,000 firm-years. The probability of selection is proportional to the firm's market cap as a percent of total market cap at the start of that month. "Crisis" periods are defined as any month in which the annualized standard deviation of daily market returns in the previous month was greater than 25%. The maximum loss is capped at 99%.

[5] To ensure the that differences between crisis periods and normal periods are driven by risk, we calibrate the crisis and normal period distributions to have the same mean return of 8.69%, which was the average market-cap weighted average during all periods. Without this calibration, the market-cap weighted average annual return during the 971 normal months was 8.03% and 11.41% during the 109 crisis months. We calibrate by adding 0.66% to each firm return during normal periods, and subtracting 2.72% from each firm return during crisis periods. This calibration slightly increases (e.g., for CRRA 2: the increase is 0.4 percent) the alpha needed during crises and slightly reduces the alpha required during normal times. (Note the average returns just cited in this footnote are the arithmetic mean, not the geometric mean, which is sometimes the more relevant for cumulating returns).

Alternative Assumptions

Including Safe Outside Income
If the investor instead has a safe outside income (e.g., a salary from a perfectly safe source which is uncorrelated with the market return, or perhaps government debt) the alpha estimates are as follows:

Table A1.2
By How Much Must a Risk-Averse Person Outperform the Market to Make Holding a Single Stock Worthwhile: For Investors with Different Amounts of Safe Outside Income

	Additional Alpha Required			
	Safe Outside Income			
	0%	25%	50%	75%
CRRA Coeficient = 1 (Log Utility)				
Regular Period	3.1%	2.5%	2.1%	1.8%
Crisis Period	5.0%	3.9%	3.3%	2.9%
CRRA Coeficient = 2				
Regular Period	6.5%	4.8%	4.0%	3.4%
Crisis Period	9.8%	7.1%	5.9%	5.1%
CRRA Coeficient = 3				
Regular Period	10.5%	7.3%	5.9%	5.0%
Crisis Period	14.7%	10.4%	8.4%	7.2%
CRRA Coeficient = 4				
Regular Period	15.0%	10.0%	7.9%	6.6%
Crisis Period	19.0%	13.7%	10.9%	9.2%

Notes: See Table A1.1, *supra*.

Allowing the Expected Alpha to Be Uncertain
In the baseline analysis, the investor knows that each return in the firm distribution will be increased by a given amount of alpha. If the investor is instead uncertain about how much alpha to expect this adds additional risk to making an alpha bet. In Table A1.3, we show how much more alpha is required if alpha expectations are uncertain[6]:

[6] We assume that the amount of alpha an investor will eventually realize is uniformly distributed. If this uncertainty takes a more likely form which looks similar to the normal distribution, the increase in the alpha demanded due to uncertainty is likely to be even larger for a given variance because risk averse investors strongly dislike extreme negative alpha outcomes, which are precluded in the analysis below by using the uniform distribution.

Table A1.3
By How Much Must a Risk-Averse Person Outperform the Market to Make Holding a Single Stock Worthwhile: When the Amount of Alpha is Uncertain

	Additional Alpha Required		
		Alpha Uncertain:	
	No Alpha Uncertainty	α~Unfm[.5*E(Alpha), 1.5*E(Alpha)]	α~Unfm[0,2*E(Alpha)]
CRRA Coeficient = 1 (Log Utility)			
Regular Period	3.1%	3.1%	3.1%
Crisis Period	5.0%	5.0%	5.1%
CRRA Coeficient = 2			
Regular Period	6.5%	6.5%	6.7%
Crisis Period	9.8%	9.9%	10.9%
CRRA Coeficient = 3			
Regular Period	10.5%	10.8%	12.1%
Crisis Period	14.7%	15.6%	22.9%
CRRA Coeficient = 4			
Regular Period	15.0%	16.0%	21.0%
Crisis Period	19.0%	20.7%	44.8%

Notes: See Table 1.1, *supra* . The maxmimum loss is capped at 99%.

Table 2: The Alpha Grid and Alpha Widget:

Table A2

Grid of the Additional Alpha a Risk Averse Investor Would Require to Hold Idiosyncratic Risk Under Different Conditions (% Return)

Market Risk (Annual SD) ↓ / Idiosyncratic Risk (Annual SD) →	10%	15%	20%	25%	30%	35%	40%	45%	50%
15%	0.9%	2.5%	4.1%	5.9%	7.8%	10.1%	12.5%	14.7%	16.0%
17%	1.2%	2.6%	4.1%	5.9%	7.8%	10.1%	12.3%	14.5%	15.9%
19%	1.4%	2.7%	4.1%	**5.8%**	**7.7%**	10.0%	12.1%	14.0%	15.6%
21%	1.5%	2.6%	3.9%	5.6%	7.5%	9.7%	11.8%	13.5%	15.1%
23%	1.6%	2.6%	4.0%	5.6%	7.5%	9.7%	11.7%	13.2%	14.9%
25%	1.7%	2.7%	3.9%	5.4%	7.4%	9.4%	11.4%	12.8%	14.6%
27%	1.4%	2.3%	3.5%	5.2%	7.1%	9.0%	10.8%	12.1%	14.3%
29%	1.3%	2.3%	3.5%	5.1%	6.9%	8.8%	10.2%	11.7%	14.1%
31%	1.5%	2.3%	3.6%	5.3%	6.9%	8.7%	9.9%	11.5%	14.3%
33%	1.0%	2.1%	3.3%	4.9%	6.5%	**8.0%**	**9.1%**	10.9%	13.9%
35%	1.4%	2.2%	3.5%	4.8%	6.5%	7.6%	8.8%	10.9%	13.7%
37%	1.8%	2.7%	3.7%	5.0%	6.4%	7.3%	8.7%	11.0%	14.1%

Average Ordinary Period (boxed: 19% market risk, 25%–30% idiosyncratic risk)

Average Crisis Period (boxed: 33% market risk, 35%–40% idiosyncratic risk)

In Table 2, reproduced here, we estimate more generally the amount of alpha risk averse investors will demand under a range of conditions, using assumptions similar to Table 1. We derive these estimates of the additional alpha a risk averse investor with CRRA coefficient of 2 would require using Monte-Carlo simulations based on monthly CRSP data from 1925-2016 to create distributions with the relevant levels of systematic and idiosyncratic risk. We create distributions with lower risk by restricting our sample to firms with lower risk over the following year and vice-versa for higher risk. Thus, by splitting the sample in different ways, we create a distribution of annual firm returns with standard deviations ranging from 10% to 73% (i.e., we create a separate distribution with 10% total risk, 11%, 12%,....72%, 73%).[7] Likewise, we create different distributions of annual market returns with 10% to 38% risk and then use these to estimate the additional alpha. We then calculate the additional alpha for any pair of market risk and idiosyncratic risk in a manner analogous to Table 1.[8]

For ease of presentation we present the relevant averages: thus 10% idiosyncratic represents the average of 8%, 9%, 10%, 11%, and 12%, etc. and similarly 15% market risk represents the average of 15% and 16%.

[7] To create each sample, we take all firms whose standard deviation over the year within +/- .02 of the target standard deviation and then randomly sample with replacement using market cap weights. To ensure all the distributions have the same expected return, we calibrate the annual return of each distribution to 8.69% (see note 5) by adjusting the returns above the median. (We do not adjust firms below the median because of issues with firms which, prior to adjustment, have lost nearly all their value over the year).

[8] For a given level of market and idiosyncratic risk, we use a distribution of firms with $Firm_{Total\ SD} = \sqrt{Market_{SD}^2 + Idio_{SD}^2}$. So for the market risk 21% and idiosyncratic risk of 20%, we use a distribution of firm returns with a standard deviation of 29%.

The results in the boxes do not exactly match Table 1 because the different methodology used in Table 2 produces distributions with somewhat different properties (shapes), even if though they have the same standard deviation and mean.[9]

Alpha Grid: Other Levels of Risk Aversion:

Tables A2.1, A2.2, and A2.3 present analogous analyses to Table 2 for investors with CRRA coefficients equal to 1, 3, and 4 respectively:

Table A-2.1

Grid of the Additional Alpha a CRRA= 1 Risk Averse Investor Would Require to Hold Idiosyncratic Risk Under Different Conditions (% Return)

Market Risk (Annual SD) ↓	Idiosyncratic Risk (Annual SD) →								
	10%	15%	20%	25%	30%	35%	40%	45%	50%
15%	0.5%	1.1%	1.9%	2.9%	4.0%	5.2%	6.5%	7.9%	8.9%
17%	0.5%	1.2%	1.9%	2.9%	4.0%	5.2%	6.5%	7.8%	8.9%
19%	0.6%	1.1%	2.0%	2.8%	3.9%	5.2%	6.4%	7.7%	8.7%
21%	0.5%	1.1%	1.9%	2.8%	3.8%	5.0%	6.3%	7.5%	8.5%
23%	0.5%	1.1%	1.9%	2.7%	3.8%	5.0%	6.3%	7.3%	8.4%
25%	0.6%	1.2%	1.8%	2.7%	3.8%	4.9%	6.1%	7.1%	8.3%
27%	0.5%	0.9%	1.6%	2.5%	3.6%	4.7%	5.9%	6.7%	8.1%
29%	0.4%	0.9%	1.6%	2.5%	3.5%	4.6%	5.6%	6.5%	8.1%
31%	0.4%	0.8%	1.6%	2.5%	3.4%	4.6%	5.4%	6.4%	8.3%
33%	0.1%	0.6%	1.3%	2.1%	3.1%	4.1%	4.8%	6.0%	7.9%
35%	0.1%	0.6%	1.3%	2.0%	3.0%	3.8%	4.7%	6.0%	7.9%
37%	0.7%	1.1%	1.7%	2.5%	3.4%	4.1%	5.0%	6.5%	8.5%

Average Ordinary Period (boxed: 19% row, 25%–30% columns)

Average Crisis Period (boxed: 33% row, 35%–40% columns)

Notes: We derive these estimates of the additional alpha a risk averse investor with CRRA coefficient of 1 would require using Monte-Carlo simulations based on monthly CRSP data from 1925-2016 to create distributions with the relevant levels of systematic and idiosyncratic risk. We create distributions with lower risk by restricting our sample to firms with lower risk over the next year and vice-versa for higher risk. Thus, by splitting the sample in different ways, we create a distribution of annual firm returns with standard deviations ranging from 10% to 74% (i.e., we create a separate distribution of firm returns with a 10% SD, 11%, 12%,...73%, 74%). Likewise, we create different distributions of annual market returns with SD of 15% to 38% and then use these distributions to estimate the additional alpha as in Table 1. For ease of presentation we present the relevant averages: thus 10% idiosyncratic risk represents the average of 8%, 9%, 10%, 11%, and 12%, etc. and similarly 15% market risk represents the average of 15% and 16%.

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[9] In Table 2 the distribution of the average "normal period" comes from firms whose standard deviation of returns was within +/- 2% of 33.5% (the market-cap weighted average standard deviation of an individual stock during normal periods). By contrast in Table 1, the sample comes from all firms during normal periods with a variety of standard deviations of returns, which average out (weighting by market cap) to 33.5%.

Table A-2.2
Grid of the Additional Alpha a CRRA= 3 Risk Averse Investor Would Require to Hold Idiosyncratic Risk Under Different Conditions (% Return)

Average Ordinary Period

Market Risk (Annual SD) ↓	**Idiosyncratic Risk (Annual SD) →**								
	10%	**15%**	**20%**	**25%**	**30%**	**35%**	**40%**	**45%**	**50%**
15%	1.5%	4.9%	7.2%	9.4%	12.0%	14.9%	18.0%	20.6%	22.2%
17%	2.4%	5.2%	7.1%	9.4%	11.9%	14.8%	17.7%	20.2%	21.9%
19%	3.6%	5.4%	7.1%	9.1%	11.7%	14.7%	17.4%	19.6%	21.3%
21%	4.0%	5.3%	6.8%	8.9%	11.5%	14.2%	16.9%	18.8%	20.7%
23%	4.3%	5.4%	6.9%	8.9%	11.5%	14.2%	16.8%	18.3%	20.4%
25%	4.3%	5.4%	6.9%	8.9%	11.4%	13.9%	16.2%	17.7%	19.9%
27%	4.0%	4.9%	6.4%	8.5%	11.0%	13.3%	15.4%	16.8%	19.4%
29%	3.8%	4.9%	6.5%	8.5%	10.8%	13.1%	14.6%	16.3%	19.0%
31%	4.1%	5.1%	6.8%	8.8%	10.8%	12.9%	14.1%	16.0%	19.1%
33%	3.9%	5.2%	6.6%	8.6%	10.5%	12.1%	13.2%	15.3%	18.5%
35%	4.5%	5.5%	7.1%	8.5%	10.5%	11.5%	13.0%	15.3%	18.1%
37%	4.7%	5.7%	7.0%	8.5%	9.9%	10.8%	12.5%	14.9%	18.2%

Average Crisis Period

Notes: We derive these estimates of the additional alpha a risk averse investor with CRRA coefficient of 3 would require using Monte-Carlo simulations based on monthly CRSP data from 1925-2016 to create distributions with the relevant levels of systematic and idiosyncratic risk. We create distributions with lower risk by restricting our sample to firms with lower risk over the next year and vice-versa for higher risk. Thus, by splitting the sample in different ways, we create a distribution of annual firm returns with standard deviations ranging from 10% to 74% (i.e., we create a separate distribution of firm returns with a 10% SD, 11%, 12%,...73%, 74%). Likewise, we create different distributions of annual market returns with SD of 15% to 38% and then use these distributions to estimate the additional alpha as in Table 1. For ease of presentation we present the relevant averages: thus 10% idiosyncratic risk represents the average of 8%, 9%, 10%, 11%, and 12%, etc. and similarly 15% market risk represents the average of 15% and 16%.

Table A-2.3
Grid of the Additional Alpha a Risk Averse CRRA =4 Investor Would Require to Hold Idiosyncratic Risk Under Different Conditions (% Return)

Market Risk (Annual SD) ↓ / **Idiosyncratic Risk (Annual SD) →**	**10%**	**15%**	**20%**	**25%**	**30%**	**35%**	**40%**	**45%**	**50%**
15%	2.5%	8.6%	11.3%	13.5%	16.3%	19.5%	23.1%	25.9%	27.4%
17%	4.8%	9.2%	11.0%	13.4%	16.1%	19.5%	22.6%	25.2%	27.0%
19%	7.5%	9.5%	11.1%	13.0%	15.8%	19.1%	22.1%	24.3%	26.2%
21%	8.1%	9.3%	10.6%	12.8%	15.6%	18.6%	21.5%	23.3%	25.4%
23%	8.3%	9.3%	10.7%	12.9%	15.6%	18.6%	21.3%	22.7%	25.0%
25%	8.3%	9.3%	10.7%	12.8%	15.5%	18.2%	20.6%	22.0%	24.4%
27%	8.0%	8.8%	10.2%	12.5%	15.1%	17.5%	19.5%	21.0%	23.6%
29%	7.7%	8.7%	10.4%	12.4%	14.9%	17.3%	18.6%	20.4%	23.0%
31%	8.0%	8.9%	10.8%	12.9%	14.9%	17.0%	18.0%	20.1%	23.0%
33%	7.9%	9.3%	10.7%	12.7%	14.7%	16.2%	17.3%	19.4%	22.3%
35%	8.6%	9.6%	11.2%	12.7%	14.6%	15.5%	17.1%	19.3%	21.8%
37%	8.5%	9.6%	10.8%	12.4%	13.6%	14.4%	16.2%	18.3%	21.4%

Average Ordinary Period: 19% market risk, 25%–30% idiosyncratic risk (boxed: 13.0%, 15.8%)

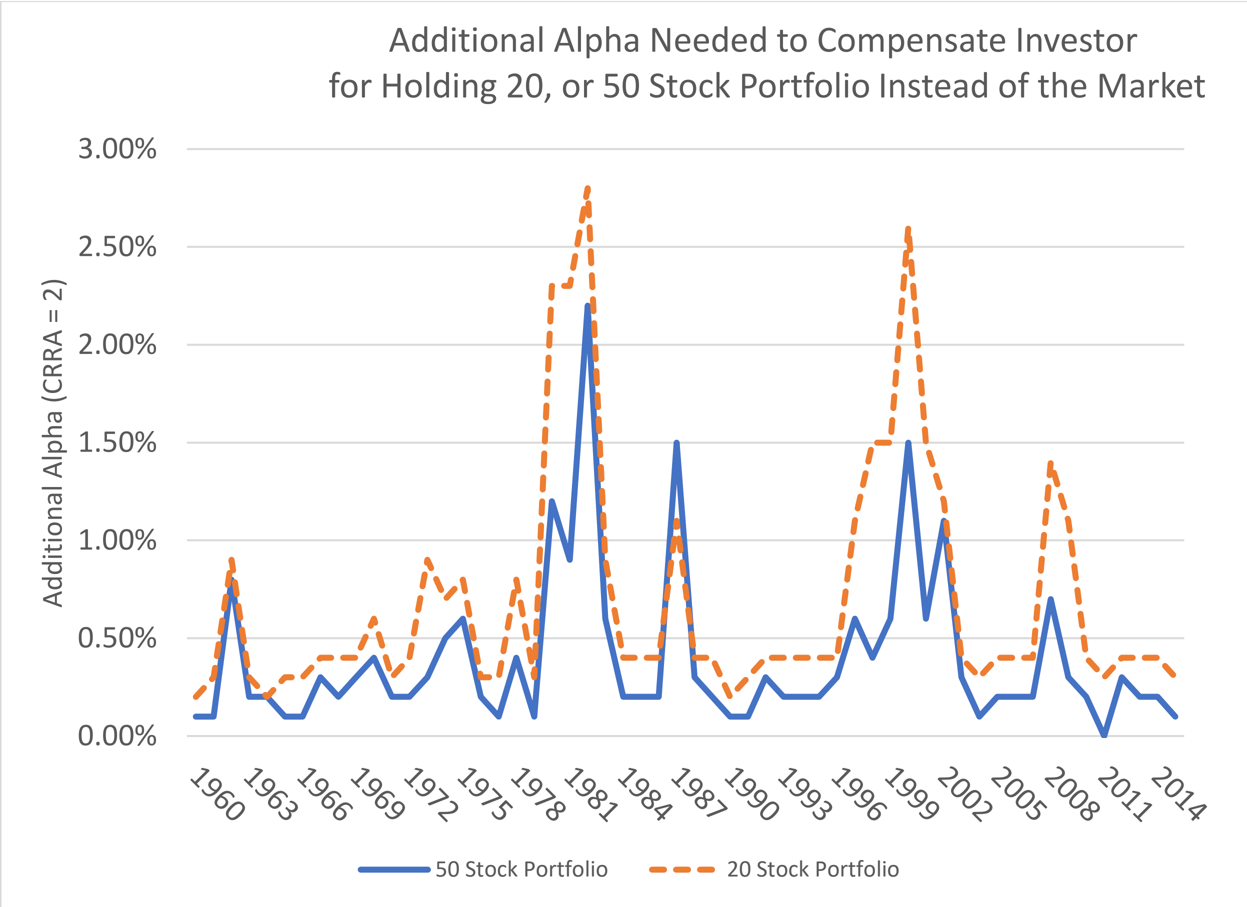
Average Crisis Period: 33% market risk, 35%–40% idiosyncratic risk (boxed: 16.2%, 17.3%)

Notes: We derive these estimates of the additional alpha a risk averse investor with CRRA coefficient of 4 would require using Monte-Carlo simulations based on monthly CRSP data from 1925-2016 to create distributions with the relevant levels of systematic and idiosyncratic risk. We create distributions with lower risk by restricting our sample to firms with lower risk over the next year and vice-versa for higher risk. Thus, by splitting the sample in different ways, we create a distribution of annual firm returns with standard deviations ranging from 10% to 74% (i.e., we create a separate distribution of firm returns with a 10% SD, 11%, 12%,...73%, 74%). Likewise, we create different distributions of annual market returns with SD of 15% to 38% and then use these distributions to estimate the additional alpha as in Table 1. For ease of presentation we present the relevant averages: thus 10% idiosyncratic risk represents the average of 8%, 9%, 10%, 11%, and 12%, etc. and similarly 15% market risk represents the average of 15% and 16%.

20 and 50 Stock Portfolios

In Figure 7 of the main text we graph the alpha required to compensate an investor with CRRA=2 for partially diversifying by owning a 20 or 50 stock portfolio which is randomly selected (weighting by market cap). Figure 7 is displayed again here as Figure A1

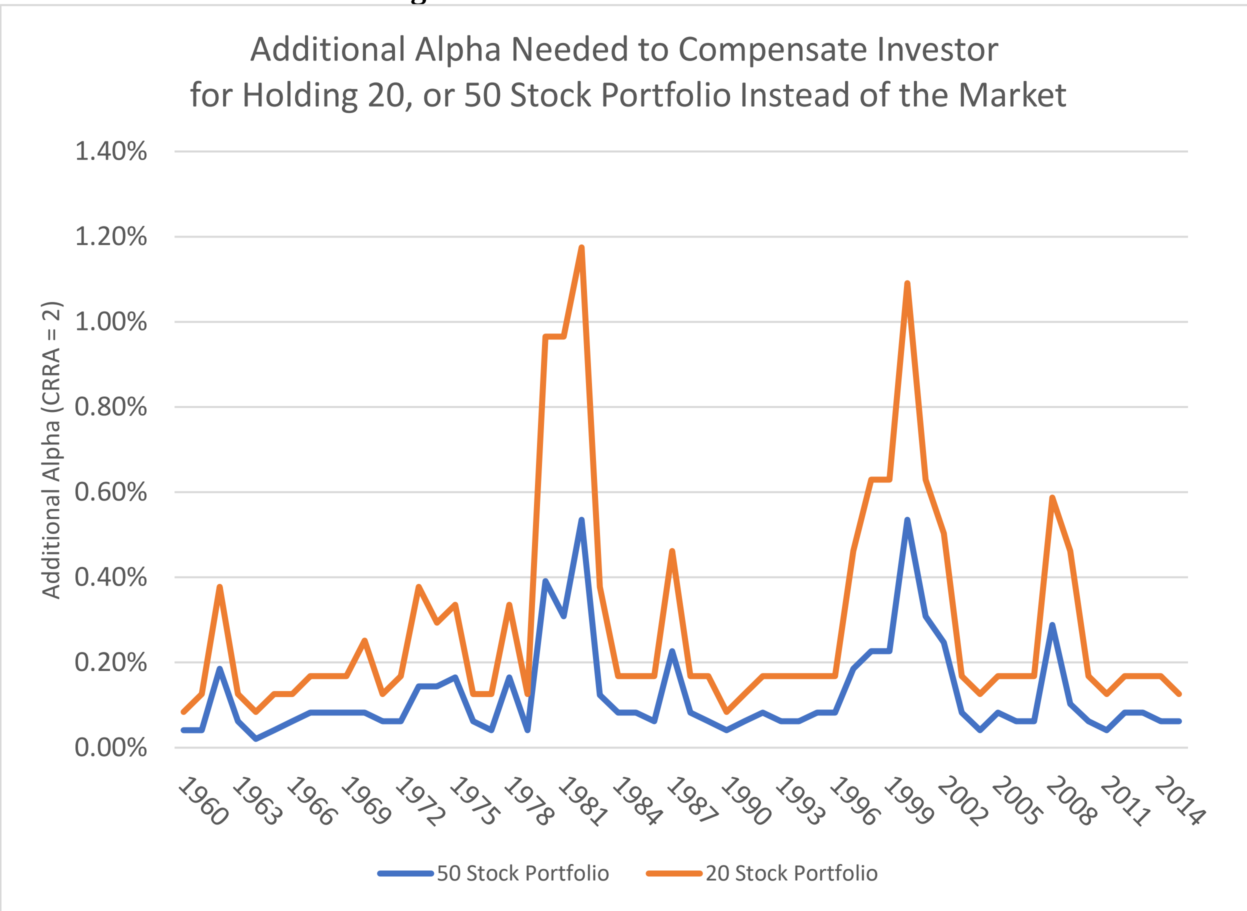
Figure A1



As noted in the main text, we arrive at Figure A1 by (1) estimating how much additional idiosyncratic risk remains in the 20 and 50 stock portfolios and then (2) converting this idiosyncratic risk (when paired with data on systemic risk in that year) into an alpha figure using the data underlying Table 2. This may, in some cases, overstate the alpha investors would actually demand to hold these portfolios. In particular, using Table 2's data implicitly assumes that the 20 and 50 stock portfolios are equally "thick-tailed" as single stocks with the same total risk.

If by contrast we assume all the distributions including those of individual firms, the 20 and 50 stock portfolios, and the market as a whole are normally distributed, the idiosyncratic risk remaining in the partially diversified 20 and 50 stock portfolios produces relatively smaller offsetting alphas.

Figure A1.2
Assuming All Relevant Distributions are Normal



There is in fact a substantial literature showing that stock returns are fatter tailed than the normal distribution. Even here, however, the remaining alphas are frequently substantial, often exceeding 50 basis points for 50 stocks.