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Optimal voting rules for two-member tenure committees

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Abstract A tenure committee first votes on whether to hire a candidate; if it does, it receives an informative performance signal, and then votes on whether to tenure the candidate; rejection at either stage returns the committee to a candidate pool, endogenising the value of the outside option. A candidate's fate depends only on the behaviour of two 'weather-vane' committee members. Committee members may vote against favoured candidates if the weather-vane is opposed; enthusiastic assessments by one of these weather-vanes may harm a candidate's chances by increasing others' thresholds for hiring him; sunk time costs may lead voters who voted against hiring to vote for tenuring him, even after a poor probationary performance. For two member committees that are patient and perceptive, the optimal voting rule is a (weak) majority at the hiring stage and unanimity at the tenure stage; when such committees are impatient or imperceptive, the double (weak) majority rule is optimal. Perversely, the performance of a patient, imperceptive committee improves as its perceptiveness further declines. Consistent with practice, falling threshold rules are not optimal. Results on optimal voting rules are also presented in limit cases as committee members' beliefs become more correlated. Finally, we compare the model to a discrete-time European options model.

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1 Introduction

Many options are adopted only after an initial commitment is made to consider them in greater depth; further, rejection of a particular option—either before or after deeper consideration—typically does not end the decision-making process, but leads to consideration of a new option. When a committee is responsible for making such decisions, the value of the decision-making process depends on the voting rules that it uses. In these problems, named *tenure committee* problems after our leading example, it is natural to ask what voting rules such committees should follow. We characterise equilibrium voting behaviour in this environment and, for committees of two members, determine the voting rule that maximises the value of the process.

To make the analysis more concrete, we exposit it in the context of the right to offer an employee a permanent position. Tenure in academia and 'up or out' rules in law, accountancy and the military are all examples of employment decisions that fall within the scope of our analysis (Meyer 1992).[1]

Portfolios of real or financial options may also be managed by teams, particularly in the non-investment community (e.g. pension and insurance companies, and commercial real estate). Bills in parliamentary democracies typically must survive two votes before being enacted into law; bicameral systems often require that bills be passed by both houses in sequence. Judicial decisions motivated Condorcet's interest in committee decisions: the possibility of appeal and penalty phases introduces a second decision-making stage in both the cases.[2] A period of engagement often precedes marriage which, if unsuccessful, typically ends candidacy. While differing in details, each of these situations involves the management of an option by committee.

To focus on the intertemporal issues associated with options management we abstract from many intratemporal ones, including incomplete information between committee members. More specifically, our committee considers a single candidate, who is either of high or low quality. However, prior to receipt of any further information, their opinions of the candidate's quality may differ. Committee members vote either for or against hiring the candidate. If hired, the candidate's performance is publicly observed, allowing members to update their beliefs about him and vote on granting him tenure. If granted tenure, the candidate's type becomes known, and value realised.

Rejecting the candidate at either stage returns the committee to the candidate pool. Before drawing a new candidate, each committee member expects to receive V, the expected game's value. As this is determined endogenously, as a function of the voting rule, a fixed point is created: the process' value depends on committee members' votes; those votes, in turn, depend on the value that committee members ascribe to rejecting a candidate and returning to the pool.

1 McPherson and Schapiro (1999) provides a recent introduction to the tenure literature, which includes Chatterjee and Marshall (2003), Chen and Ferris (1999), Ehrenberg et al. (1998), Carmichael (1988) and Ito and Kahn (1986). Another academic example is the practice of refereeing articles, which are often accepted for publication only after surviving an initial round of refereeing.

2 Socrates' trial provides history's most notorious penalty phase decision. Athenian juries first voted on the defendant's innocence or guilt; a guilty finding then led to a second vote over the proposed penalties. Thus, Socrates was found guilty by 280 jurists, but 360 voted for his execution.

As committee members' priors are common knowledge, we model common values with 'open disagreement' (van den Steen 2004).[3] Although voting is binary, the tenure committee problem differs from the classic environment of May (1952) both in its two-stage voting and in endogenising the value of the outside option. These features remove the equivalence between voting rules and social welfare preorderings, and with it the expectation that a simple majority rule is optimal (q.v. Moulin 1994).

Theorems 1 and 2 present equilibrium voting strategies for general committees of size N taking V as given. Intrapersonally, we identify a tenure 'weather-vane' voter—the member who always votes with the winning side in the tenure vote. Other committee members then condition their votes at the hiring stage on their own priors and the tenure weather-vane's. As a probationary weather-vane may also be identified, a sufficient statistic for the candidate's fate is the priors of these two committee members.

The equilibrium voting behaviour provides three take-home lessons about how the tenure committee problem differs from that of a single decision maker in a two-stage problem. First, committee members may vote against hiring a personally favoured candidate if the tenure weather-vane is sufficiently opposed to ensure his defeat at the tenure stage. Second, at the other extreme, a sufficiently enthusiastic tenure weather-vane can cause committee members to apply higher standards to their own hiring votes.[4] Third, time sunk during the probationary process can lead a committee member who opposed hiring a candidate to support tenure in spite of a bad probationary performance. All these conclusions form testable predictions about how the tenure structure influences voting behaviour.

Having considered the case of exogenous V, the article then turns to the fixed point problem associated with endogenous V. As each additional committee member adds a dimension of integration to the problem, we specialise to the case of $N = 2$ to derive explicit representations for the value functions.[5]

Comparing the value of each rule at a given level of patience and perceptiveness then allows derivation of optimal voting rules. When committees are patient or perceptive (in the sense of receiving more informative signals during the probationary period), the rising threshold rule—a (weak) majority at hiring but unanimity at tenure—is optimal.[6] Otherwise, for committees that are both impatient and imperceptive, the double (weak) majority rule—a single vote to pass at each stage—is optimal; this rule allows impatient or imperceptive committees to reduce the costs of waiting for a

[3] An earlier discussion article (Ayres et al. 2007) presents an interpretation of the model as one with common priors, but private values.

[4] This form of non-monotonicity differs from May's, in which more votes for a candidate would harm the candidate. In an information aggregation environment, the 'supermajority penalty' rule of Chwe (2007) is non-monotonic in May's sense.

[5] We initially analyse the N member committee, instead of specialising to $N = 2$, as doing so allows us to derive general results at almost no further cost.

[6] The rising threshold result resembles that in Meyer (1991, Sect. 7), where it may be optimal to bias against the winner of an earlier contest. Here, discounting and aggregation via the committee work in opposite directions: an individual committee member lowers her standards, being more willing to accept a candidate once the probationary time costs are sunk; the committee, as a whole, however, has a higher standard for the second vote.

probationary signal as voting is such that hiring leads automatically to tenure, making V independent of perceptiveness as the signal is ignored. The double unanimity and falling threshold (unanimity followed by weak majority) voting rules are dominated for all parameter values (Theorems 3 and 4).[7]

These optimal rules are both intuitively appealing, and suggest a generalisation to larger committees: patient or perceptive committees should hire more readily than they grant tenure, as hiring entails either little cost (patience) or provides good information (perception) about a candidate; impatient, imperceptive committees should be undemanding at both the stages, as delay in appointing a candidate is expensive (impatience) and probation provides little information (imperception). Our analysis also suggests the following testable predictions: falling thresholds should never be observed, or should rules requiring unanimity at both the stages, and a comparative statics result whereby committees which become more patient or perceptive should increase the strictness of their tenure stage voting rules.

These predictions about optimal voting rules seem broadly consistent with anecdotal evidence on existing voting rules, in that tenure systems with double unanimity or falling thresholds are rare, at best. Moreover, our comparative statics result may help explain the variance of voting rules across institutions. Within a given country, the tenure decision is more stringent in top departments, consistent with our theory: a top department will be a better judge of new talent (perceptiveness), making the rising threshold rule optimal if it is also sufficiently patient; such departments that do not adopt the rising threshold rule will be at a competitive disadvantage, introducing evolutionary pressure to adopt the rule. Across countries, the tenure decision is regarded as generally more stringent within the US.[8] This may reflect the prevalence of top departments in the US. Otherwise, it would seem to require US universities to be generally more patient or perceptive than their foreign counterparts; while this may be consistent with an unusually large private university sector in the US, it is a matter of speculation whether this structure would have the required effect: does greater independence from government harden budget constraints and property rights?

Given an optimal voting rule, the models' statics are also intuitively appealing: the value of a post generally increases in a committee's patience and perceptiveness. One comparative static result is peculiar: patient but imperceptive committees' performance improves slightly as their perceptiveness falls. This owes to the more optimistic committee member's willingness to 'take a chance' on a candidate under the rising threshold rule. As the signal's informativeness declines, the optimist loses hope that the pessimist will be swayed by a good probationary performance. The optimist thus takes fewer such chances.

[7] Without private information to pool, the former is dominated for reasons other than those developed in Feddersen and Pesendorfer (1998). Their jurists, who share common interests, may convict more innocent defendants under unanimity than under majority votes: a pivotal jurist who initially believes a defendant innocent will allow guilty votes from the remaining jurists to overrule her private signal. Under weaker rules, the jurist does not have this incentive to vote against her private signal.

[8] Prüfer and Walz (2009) claim that European universities often operate on the basis of consensus, and argue that majority rule would likely increase faculty welfare. This is consistent with our finding that a double consensus rule is never optimal.

Finally, we consider the consequences of allowing committee members' prior beliefs about a candidate to be correlated. In the extreme case of perfect correlation, the choice of voting rule becomes immaterial as committee members vote identically (Theorem 6). In this case, there is effectively a single committee member and, for all levels of patience and perceptiveness, value exceeds that under uncorrelated views: compromising among conflicting views is costly, in contrast to information aggregation models of committee decisions. For intermediate values of correlation, committees for whom the probationary signal is useless should switch from the double majority rule to the rising threshold rule at lower levels of patience as their priors become more correlated: while increased correlation decreases the benefits of taking the second committee member's beliefs into account, it decreases the time costs even more so. Again, these predictions on belief correlation form testable predictions.

There is a small literature on optimal voting rules in the context of strategic information aggregation problems (see Chwe 2007 for a review). There have also been few analyses of two-stage committee decisions. One exception is Manzini and Mariotti (2006), which takes each committee's (rational) preference ordering as given to focus on the rationality properties of their composition. Polborn (2000) analyses an environment in which voters decide on tax reform voting rules when young (and poor); after a stochastic shock, when they are older (and wealthier) they vote on tax reform.

Section 2 presents our model. Section 3 analyses voting when V is taken as given. Section 4 then solves the fixed point problem for V for a two member committee. Section 5 compares the tenure committee problem to a standard options problem. Section 6 concludes. The appendix contains the proofs that use techniques from algebraic geometry, which are also introduced there. Finally, code used in support of the article is available online.[9]

2 The model

Consider a pool of ex ante identical candidates.[10] In the game's prehistory, one of them is shortlisted and presented to a committee of N risk neutral members, indexed by i. For expositional clarity, assume the committee members to be female and the candidate male.

The candidate will either be good for the department ($\tau = 1$) or bad ($\tau = 0$) if tenured. While committee members share common interests, their assessments of the candidate vary. The subjective probability that each assigns to the candidate being a good hire is $p_{0i} \in [0, 1]$ with $p_{01} > \cdots > p_{0N}$.[11] Finally, these priors are common knowledge.

At time 0, the committee members vote on hiring. Voting is costless and mandatory: abstentions are not allowed. If strictly more than $\delta_0 N$ of committee members approve, the candidate is given probation.

[9] www.socscistaff.bham.ac.uk/rowat/research/ARZ-code.zip.

[10] This model differs from those that allow an agent to specify the order in which candidates are tested; q.v. Weitzman (1979).

[11] The assumption of strict inequalities simplifies analysis while retaining genericity.

If the candidate is hired, his probationary performance emits a public signal, $\theta \in \{0, 1\}$. It is common knowledge that the signal accurately reflects the candidate's type with probability $\sigma : P(\theta = 1|\tau = 1) = P(\theta = 0|\tau = 0) = \sigma$.[12] Without loss of generality, $\sigma \in (\frac{1}{2}, 1]$: otherwise, the signal's complement could be used as the informative signal.

Thus,

$$E_{0i}[\theta] = \sigma p_{0i} + (1 - \sigma)(1 - p_{0i}); \quad (1)$$

where E_{0i} are member i's expectations at time 0.

After the signal's receipt, committee members calculate p_{Ti}, their posterior beliefs:

$$p_{Ti}(\theta, p_{0i}) = \frac{P(\theta|\tau = 1) p_{0i}}{P(\theta|\tau = 1) p_{0i} + [1 - P(\theta|\tau = 1)](1 - p_{0i})}. \quad (2)$$

Thus $p_{T1} > \cdots > p_{TN}$.

At time $T > 1$, a tenure vote is taken. If strictly more than $\delta_T N$ committee members approve, the candidate is tenured.[13] As this date cannot be brought forward, the real option is European rather than American.

If, at either vote, the candidate fails, he is dismissed and the department returns to the pool of candidates. If a candidate is not hired, the department returns to the pool the following year. The value of restarting the hiring process is βV, where $\beta \in [0, 1]$ is the discount factor. If the candidate is hired but denied tenure, the department immediately returns to the pool, expecting to receive value $\beta^T V$. It neither suffers costs nor gains benefits from having granted probation to a candidate. Finally, if a candidate is granted tenure, committee member i anticipates an expected payoff of $\beta^T p_{Ti}$: the actual payoff is equal to the candidate's type. This pattern of play is summarised in Fig. 1. Payoffs are discounted to time 0.

That the probationary period does not impose costs or deliver benefits in its own right may be interpreted as the probate's teaching load financing the probationary period. In options parlance, no dividends are paid. Granting tenure to a $\tau = 0$ candidate may be similarly interpreted. (As tenure does not convey salary guarantees, this may be thought of as adjusting a tenured, non-research productive faculty member's salary.) This structure may fit the legislative process more closely: a bill does not impose costs or benefits until it is enacted, but information about these is gained during its consideration.

For some parameter values, the tree in Fig. 1 degenerates: some terminal nodes are dominated for all priors. For example, where it possible that $\sigma = \frac{1}{2}$, refusing tenure to a candidate granted probation would always be dominated by not hiring the candidate. Similarly, $T = \infty$ would leave tenure refusal dominated by initial rejection. Finally, if $\beta = 1$, all candidates are hired: hiring allows no worse a payoff to be obtained

[12] A richer signal function would allow $\sigma_1 \equiv P(\theta = 1|\tau = 1) \neq \sigma_0 \equiv P(\theta = 0|\tau = 0)$. In return for complicating analysis, this second parameter would change the sensitivity of posterior beliefs to the signal.

[13] We address the fixed point problem, below, by setting $T = 2$. Until then, we develop the exposition for the fully general $T > 1$ as this allows the effect of the probationary period to be explicitly seen.

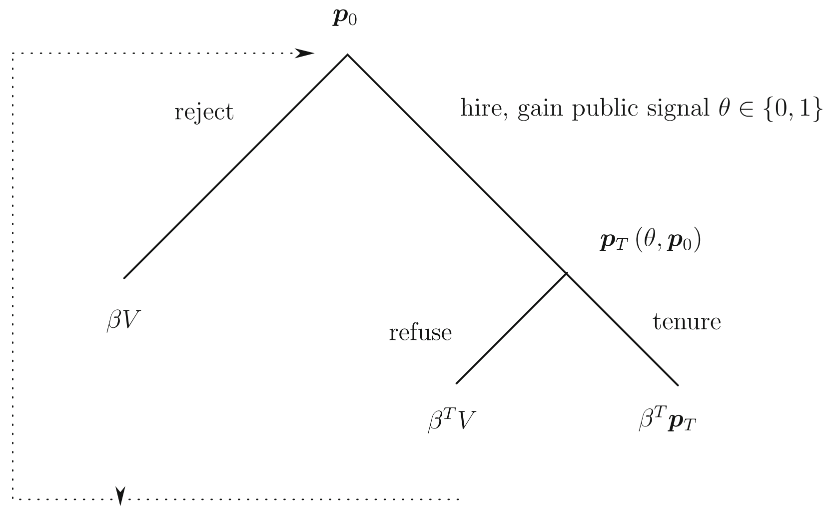

Fig. 1 The committee's decision tree

than that possible by returning to the pool; a positive signal, however, convinces any committee member that the candidate is worth tenuring.

It follows that $\beta < 1$ implies that $V(\cdot) \in [0, 1)$: even if a $\tau = 1$ candidate is discovered immediately, it takes time to tenure him. The lower bound is achieved whenever the terminal node is reached with a $\tau = 0$ candidate tenured. If a perfectly patient committee never granted tenure, the indeterminate $V = V$ would result.

At this point, V is treated as constant. Fixed point arguments will later be used to compute V as a function of its parameters.

Now define strategies:

Definition 1 A *strategy* for committee member i is a pair of functions $v_{0i} : [0, 1]^N \rightarrow \{0, 1\}$ and $v_{Ti} : [0, 1]^N \times \{0, 1\}^N \rightarrow \{0, 1\}$.

Strategies map to votes: action 1 corresponds to a 'yes' vote, and action 0 to a 'no'. The hiring vote maps from priors, while the tenure vote maps from posteriors and past votes.

The expected utility of member i, as assessed at time T and discounted to time 0, is:

$$u_{Ti}(\boldsymbol{v}_T) \equiv \left\{ \begin{array}{l} \beta^T p_{Ti} \text{ if } \sum_{j=1}^{N} v_{Tj} > \delta_T N \\ \beta^T V \text{ otherwise} \end{array} \right\}.$$

Her expected utility at time 0 is

$$u_{0i}(\boldsymbol{v}_T, \boldsymbol{v}_0) \equiv \left\{ \begin{array}{l} u_{Ti}(\boldsymbol{v}_T) \text{ if } \sum_{j=1}^{N} v_{0j} > \delta_0 N \\ \beta V \text{ otherwise} \end{array} \right\}.$$

Definition 2 A strategy profile $\boldsymbol{v}^* = \left\{v_{0i}^*, v_{Ti}^*\right\}_{i=1}^N$ and posterior beliefs $\boldsymbol{p}_T(\boldsymbol{p}_0, \theta)$ constitute a *voting equilibrium* iff, $\forall i = 1, \ldots, N$:

- $\boldsymbol{v}^*$ contains no weakly dominated strategies;
- $v^*_{Ti} \in \arg\max_{\{0,1\}} \Big\{ P\left(\sum_{j\neq i} v^*_{Tj} + v_{Ti} > \delta_T N\right) \beta^T p_{Ti} + \left[1 - P\left(\sum_{j\neq i} v^*_{Tj} + v_{Ti} > \delta_T N\right)\right] \beta^T V \Big\}$;
- $v^*_{0i} \in \arg\max_{\{0,1\}} \Big\{ P\left(\sum_{j\neq i} v^*_{0j} + v_{0i} > \delta_0 N\right) u_{Ti}(\boldsymbol{v}_T) + \left[1 - P\left(\sum_{j\neq i} v^*_{0j} + v_{0i} > \delta_0 N\right)\right] \beta V \Big\}$;
- $p_{Ti}(p_{0i}, \theta)$ is defined by Eq. 2.

Thus, a *voting equilibrium* is a restricted Perfect Bayesian equilibrium with trivial learning. Eliminating weakly dominated strategies reduces the effective domain of v_{Ti} to the support of p_{Ti}: committee members vote 'sincerely' at time T (q.v. Moulin 1994). As dominated strategies cannot be present in the support of mixed strategies, this allows concentration on pure strategies at time T. Finally, it ensures a unique equilibrium in the time T stage game (see Theorem 1), simplifying time 0 analysis. It will be seen later that this effectively makes the domain of v_{0i} two dimensional. While it is natural to consider coalitions in the context of small committees (q.v. Peleg 2002), this article leaves that as an open question.

3 The meetings

3.1 The tenure committee meeting

Perfection allows v_{Ti} to be considered independently of v_{0i}:

Theorem 1

$$v^*_{Ti}(p_{Ti}) = \left\{ \begin{array}{l} 1 \text{ if } p_{Ti} \geq V \\ 0 \text{ otherwise} \end{array} \right\} \forall i \in N.$$

Proof When committee member i is pivotal, she chooses the terminal payoff that she judges higher. Otherwise, her choice is without consequence. Voting against her preferred terminal payoff is weakly dominated.

Therefore,

Definition 3 Member k is a *weather-vane voter at time* T for beliefs $\boldsymbol{p}_T$ and voting rule δ_T when

$$k = \begin{cases} \lceil \delta_T N \rceil \text{ if } \delta_T N \text{ is not an integer;} \\ \lceil \delta_T N \rceil + 1 \text{ otherwise;} \end{cases} \tag{3}$$

where $\lceil \cdot \rceil$ is the least integer function.

A weather-vane voter is therefore a δ_T-percentile voter, a generalised median voter. If $\delta_T N$ is not an integer, then $\lceil \delta_T N \rceil > \delta_T N$ and there are enough votes for passage. If $\delta_T N$ is an integer, then $\lceil \delta_T N \rceil = \delta_T N$ which, as we use strict inequality, is insufficient for passage; $\delta_T N + 1$ suffices.

Thus, the committee's decision coincides with the weather-vane's vote. A weather-vane differs from a dictator in that the aggregation rule does not privilege her ex ante or independently of others' beliefs. To see how a weather-vane and a pivotal voter differ, consider:

Example 1 Let $N = 5, \delta_T = \frac{1}{2}$. Therefore, $k = 3$. Suppose further that $p_{05} \geq V$: even the most sceptical committee member votes for tenure. Then k is not pivotal. Now suppose that the priors are such that members 4 and 5 will vote against tenure. In this case, the remaining members are all pivotal.

The weather-vane exists as a consequence of mandatory binary voting and the elimination of weakly dominated strategies. Generically, the weather-vane is unique.

3.2 The hiring committee meeting

Now consider voting at the hiring committee meeting. Again, each committee member compares her priors to threshold levels. Now, as each member seeks to predict the candidate's fate if he is brought to tenure, the thresholds depend on the tenure weather-vane's priors as well.

Define

$$\bar{p} \equiv \frac{\sigma V}{\sigma V + (1-\sigma)(1-V)}; \tag{4}$$

$$\underline{p} \equiv \frac{(1-\sigma)V}{(1-\sigma)V + \sigma(1-V)}. \tag{5}$$

Thus, $\bar{p}$ (respectively $\underline{p}$) leaves committee member i indifferent between returning to the candidate pool next year and granting the existing candidate tenure if he produces a bad (respectively good) performance signal. If $p_{0i} \geq \bar{p}$ (respectively $p_{0i} \leq \underline{p}$) then committee member i will vote for (respectively against) tenure even when the candidate emits the bad (respectively good) signal during probation:

Lemma 1

$$p_{0i} \geq \bar{p} \Leftrightarrow p_{Ti}(0, p_{0i}) \geq V;$$
$$p_{0i} \leq \underline{p} \Leftrightarrow p_{Ti}(1, p_{0i}) \leq V.$$

Proof By Eq. 2 and Definitions 4 and 5,

$$p_{Tk}\left(1, \underline{p}\right) = p_{Tk}(0, \bar{p}) = V. \tag{6}$$

The result follows from the monotonicity of p_{Ti} in p_{0i}. □

Obviously, a candidate who performs badly must enjoy higher priors to be supported at the tenure meeting:

Lemma 2 $1 > \bar{p} > \underline{p}$.

Proof The first inequality follows from Eq. 4.

Assume that the second does not hold. Then there exists a $p_{0i} \in [\bar{p}, \underline{p}]$ such that, by Lemma 1, $p_{Ti}(1, p_{0i}) \leq V \leq p_{Ti}(0, p_{0i})$. As $\sigma > \frac{1}{2}$, the posterior increases in the signal, this is a contradiction. □

As priors are common knowledge, all committee members know at $t = 0$ that $p_{0k} \geq \bar{p}$ ensures that the tenure weather-vane k will vote for the candidate if he reaches the tenure meeting. Refer to candidates so hired as *permanent hires*. Similarly, $p_{0k} < \underline{p}$ assures them that k will oppose the candidate. Finally, when $p_{0k} \in [\underline{p}, \bar{p})$ the candidate, if hired, is a *probationary hire*, whose probability of winning tenure is assessed by committee members to be $E_{0i}(v^*_{Tk}) = \sigma p_{0i} + (1-\sigma)(1-p_{0i})$.

Now define

$$\tilde{p} \equiv \beta^{1-T} V; \tag{7}$$

the initial belief that leaves i indifferent between returning to the pool and hiring the candidate when she knows that the weather-vane will support the candidate's tenure ($p_{0k} \geq \bar{p}$) regardless of his performance. Thus, $\tilde{p}$ is independent of σ. The definition is derived from

$$E_{0i}\left[\beta^T p_{Ti}\left(\theta, \tilde{p}\right)\right] = \beta V.$$

As the committee members are rational and $\boldsymbol{p}_0$ is common knowledge, $E_{0i}[\cdot] = E_0[\cdot] \forall i \in N$.

Now define the expected payoff to hiring a candidate if $p_{0k} \in [\underline{p}, \bar{p})$, so that the weather-vane only votes for tenure if $\theta = 1$:

$$\begin{aligned} f\left(p_0\right) &\equiv P\left(\theta = 1\right) \beta^T p_T\left(1, p_0\right) + P\left(\theta = 0\right) \beta^T V \\ &= \left\{\sigma p_0 + \left[\left(1-\sigma\right) p_0 + \sigma\left(1-p_0\right)\right] V\right\} \beta^T. \end{aligned} \tag{8}$$

Lemma 3 *The function* $f(p_0)$ *is strictly increasing over* $(0, 1)$.

Proof

$$f' = [\sigma + (1-2\sigma)V]\,\beta^T;$$

so that $f' > 0 \Leftrightarrow V < \frac{\sigma}{2\sigma-1}$. This last term monotonically decreases from infinity at $\sigma = \frac{1}{2}$ to $1 \geq V$ at $\sigma = 1$. □

Finally, let $\hat{p}$ be the initial belief that leaves i indifferent between returning to the pool and probationarily hiring the candidate, so that $f(\hat{p}) = \beta V$. As the performance signal is now used, $\hat{p}$ is a function of σ:

$$\hat{p} \equiv \frac{\beta^{1-T} - \sigma}{(1-\sigma)V + \sigma(1-V)} V. \tag{9}$$

The analysis of $\frac{\sigma}{2\sigma-1}$ in the proof of Lemma 3 shows that $\hat{p}$ is finite.

We expect $\hat{p} \geq \underline{p}$ as there would otherwise be a range of priors in which members were willing to hire even if the candidate stands no chance of obtaining tenure. This is easily confirmed: $f(\underline{p}) = \beta^T V \leq \beta V = f(\hat{p})$. Lemma 3 establishes the inequality. The inequality reduces to an equality as $\beta \to 1$.

Voter i's optimal behaviour therefore depends on p_{0i} and p_{0k}:

Theorem 2

$$v_{0i}^*(p_{0i}, p_{0k}) = \begin{Bmatrix} 1 \text{ if } (p_{0i}, p_{0k}) \geq (\tilde{p}, \bar{p}) \ or \ (p_{0i}, p_{0k}) \in \left[\hat{p}, 1\right] \times \left[\underline{p}, \bar{p}\right) \\ 0 \text{ otherwise} \end{Bmatrix} \forall i \in N.$$

Figure 2 illustrates and explains the results:

- I Member i knows that, if hired, the candidate will fail tenure. As this delays the department's return to the pool, she opposes hiring, even if she is already convinced that the candidate should be granted tenure.
- II Member i knows that, if hired, the candidate will gain tenure regardless of his performance. As she is sufficiently sceptical about the candidate, she opposes the hire. When $p_{0i} \in (\hat{p}, \tilde{p})$ she votes against him even though she would support him were the tenure weather-vane less enthusiastic. Call this effect, indicated by the arrow, *inverse enthusiasm*.
- III Again, member i knows that the candidate will gain tenure if hired. Now, however, she is sufficiently confident in the candidate to take the chance of hiring him.
- IV Member i is sceptical of the candidate. She therefore votes against hiring the candidate, rather than trusting k to decide his fate.
- V Member i votes for the candidate as she is willing to allow the weather-vane to decide his fate.

The relative magnitudes of $\tilde{p}$, $\hat{p}$ and $\bar{p}$ depend, in part, on:

$$L(\beta, \sigma, T) \equiv \frac{\sigma\beta^{T-1} - (1-\sigma)}{(2\sigma - 1)} \leq 1. \tag{10}$$

Lemma 4

$$V \leq L(\beta, \sigma, T) \Leftrightarrow \hat{p} \leq \tilde{p} \leq \bar{p}. \tag{11}$$

Proof By definition of $\hat{p}$ and $\tilde{p}$, their inequality may be expressed as

$$\frac{\beta^{1-T} - \sigma}{\sigma + (1-2\sigma)V} V \leq \beta^{1-T} V.$$

As, by Lemma 3, the left-hand side of this is positive for $\sigma \in (\frac{1}{2}, 1]$, this may be manipulated for the result.

The inequality in $\bar{p}$ and $\tilde{p}$ follows similarly. □

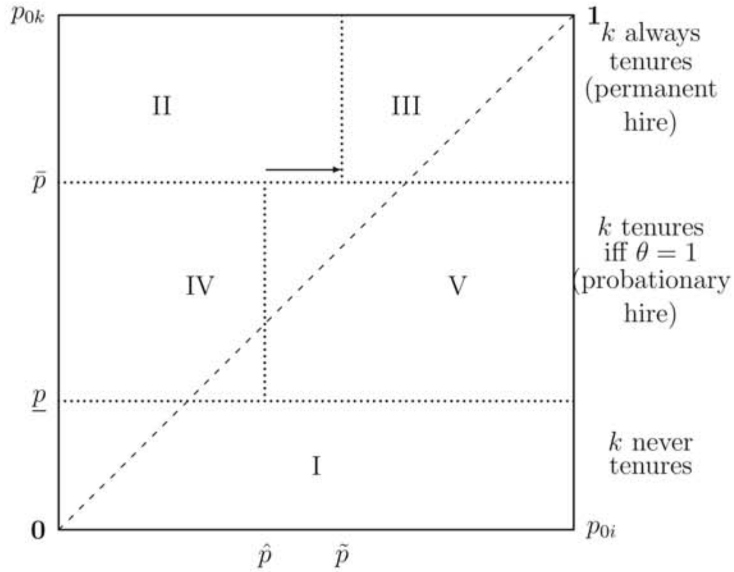

Fig. 2 Voter i's optimal behaviour at time 0 ($V < L$ case)

Thus, the inverse enthusiasm effect, whereby an enthusiastic tenure weather-vane ($p_{0k} \geq \bar{p}$) discourages other committee members from hiring a candidate ($\tilde{p} \geq \hat{p}$) holds when $V \leq L$. Otherwise, the opposite—which we term *pro-enthusiasm*—holds.

Think of $L(\beta, \sigma, T)$ as the option's strike price and V as the return to buying the market.[14] When the strike price exceeds the return to the market ($V \leq L$), i requires a higher standard to favour hiring if k will grant tenure ($p_{0k} \geq \bar{p}$): hiring exercises the option at a high price. If k is not convinced, then hiring only exercises the option in the good state of nature, a proposal more appealing to i.

Otherwise, when the return to the market exceeds the strike price ($V \geq L$), i is more easily convinced to favour hiring if she knows that k will grant tenure: hiring exercises the option at a low price. If k is not convinced, i is more easily convinced to return to the market, with its high return.

Thus, a committee member demands a higher prior to support hiring if she wants to do so on a different basis (e.g. probationary or not) than does the tenure weather-vane.

When i and k are the same committee member, her priors lie along the dotted diagonal line in Fig. 2. In this case, $V \leq L$ divides her priors into three zones: those in which she will not vote for tenure, regardless of performance; probationary hiring; and permanent hiring. On the other hand, when $V > L$, the probationary hiring zone disappears: in Fig. 2, diagonal passes through region II. In that region, the tenure weather-vane opposes hiring but will vote for tenure even after a bad probationary performance, once the time costs of reaching the tenure meeting have been sunk.

When inequality 11 holds, a weaker version of this reasoning applies in region IV: $p_{0k} \in (\underline{p}, \hat{p})$ implies that the tenure weather-vane votes against hiring, although she would vote for tenure if the candidate was hired and generated $\theta = 1$.

[14] The options interpretation is developed further in Sect. 5.

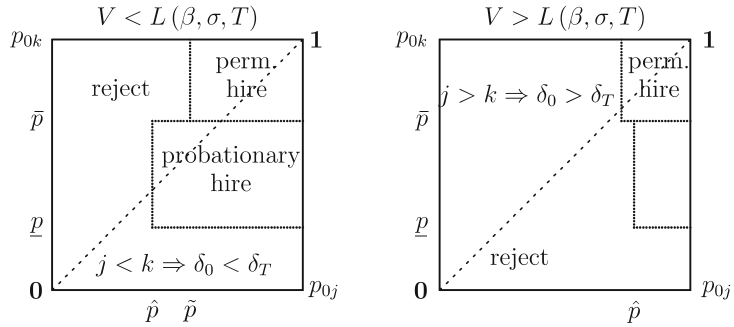

Fig. 3 Outcomes given weather-vanes j and k

As every agent i votes by comparing her p_{0i} to the same thresholds, a hiring weather-vane at $t = 0$ may be defined analogously to the tenure weather-vane at $t = T$:

Definition 4 Member j is a *weather-vane voter at time 0* for beliefs $\boldsymbol{p}_0$ and voting rule δ_0 when

$$j = \begin{cases} \lceil \delta_0 N \rceil & \text{if } \delta_0 N \text{ is not an integer;} \\ \lceil \delta_0 N \rceil + 1 & \text{otherwise;} \end{cases} \tag{12}$$

The hiring process' outcome is thus a function of the beliefs of weather-vanes j and k, as shown in Fig. 3.[15] In general, only half of this space is accessible. Which half is depends on the voting rule adopted. If, e.g. $j < k$, the tenure weather-vane, by definition, is at least as enthusiastic about a candidate as is the hiring weather-vane. In this case, the lower triangles are accessible. When $j = k$, only the diagonal line is accessible.

Candidates' success therefore depends on the beliefs of no more than two committee members, who are determined by their ordinal beliefs and the voting rule. This ordinality prevents easy expression of a candidate's success in terms of the moments of $\boldsymbol{p}_{0i}$: examples of voting rules and priors can be selected to help a candidate through higher mean priors, lower mean priors (by the inverse enthusiasm effect), decreased variance or increased variance.

Finally, the conclusions from the initial consideration of degeneracy, based on the decision tree in Fig. 1, can, in the light of Figs. 2 and 3, be stated in terms of threshold beliefs, V and L. Setting $\sigma = \frac{1}{2}$ implies $\underline{p} = \bar{p} = V > L = -\infty$, causing the possibility of probationary hiring to disappear. Setting $T = \infty$ sets $\hat{p}$ and $\tilde{p}$ to infinity as well and $L < 0$. Thus, the committee always rejects candidates. Setting $\beta = 1 \Rightarrow L = 1$, so that $L \geq V$. Thus, probationary hiring is retained. As $\sigma \to 1$, $\underline{p} \to 0$, $\bar{p} \to 1$ and $\hat{p} \to \frac{\beta^{1-T}-1}{1-V}$: in the limit, the fate of any candidate is hired will depend on his probationary performance.

[15] The promotion rate to tenure in law schools is much higher than that of other departments in major research universities (cf. Siow (1998); Chused (1988)). If discount factors do not vary significantly by discipline, this reduced use of probationary hiring could be explained if σ was lower in law schools.

While V has been treated as a parameter it depends on the committee's voting behaviour. This presents a fixed point problem: the thresholds $\hat{p}$, $\tilde{p}$, $\underline{p}$ and $\bar{p}$ depend on V which, in turn, depends on the thresholds. The next section addresses this problem.

4 Optimal voting in two member committees

This section endogenises the value of rejecting a candidate by recognising that the committee's behaviour on returning to the pool continues to be governed by the same voting rules governing its current behaviour. We make two assumptions to simplify analysis. First, assume that each committee member expects her priors over an as-yet-unknown next candidate to be independently drawn from the uniform distribution, $\boldsymbol{p}_0 \sim UID[0, 1]$. As $p_{01} > p_{02}$ (with measure one), committee members 1 and 2 can only be identified as such once their priors have been realised; refer to them, ex ante, as a and b. Without loss of generality, as committee members are identical before their priors are realised, we concentrate on b's expected payoffs.

Second, we consider only equilibria in the repeated game across candidates that do not condition on history; thus, V and the thresholds derived in the previous section are stationary, so that equilibria are as well.[16]

To indicate that value varies by voting rule, let V_{jk} be the value function under weather-vanes j and k. We now consider the four possible combinations of weather-vanes, (j, k).

4.1 Double (weak) majority: $(j, k) = (1, 1)$

The case of $(j, k) = (1, 1)$ is that of (weak) majority rule at both the stages. This corresponds to any voting rule, (δ_0, δ_T), in which $\delta_0, \delta_T \in (0, \frac{1}{2})$. The more optimistic committee member is the weather-vane at both the stages.

The diagonal in Fig. 3 then allows construction of an expression for V_{11}. By the inequalities in 11, this contains either two or three relevant intervals. Consider the simpler case first.

When the inequalities in 11 do not hold, V_{11} is bounded below by $L(\beta, \sigma, T)$. Thus, as displayed in the right panel of Fig. 4, there are only two relevant intervals: probationary hire is excluded. Below its diagonal, a is the more optimistic member, so that $p_{01} = p_{0a}$; above the diagonal, the roles are reversed and $p_{01} = p_{0b}$.

In the lower inner triangle, weather-vane a rejects the candidate, so that b expects a payoff of βV_{11}. In the lower quadrilateral, weather-vane a hires and tenures the candidate; the payoff expected by b now depends on her prior. The situation is symmetric in the upper triangle. In this case, while b still expects a payoff of $\beta^T p_{0b}$ if the candidate is tenured, this is higher than it would be were a the weather-vane.

16 As voting thresholds depend on committee members' priors, the vector of priors could be taken as a state variable, and the equilibrium defined as Markov perfect. However, as priors are independent across candidates, the Markov process implied is degenerate.

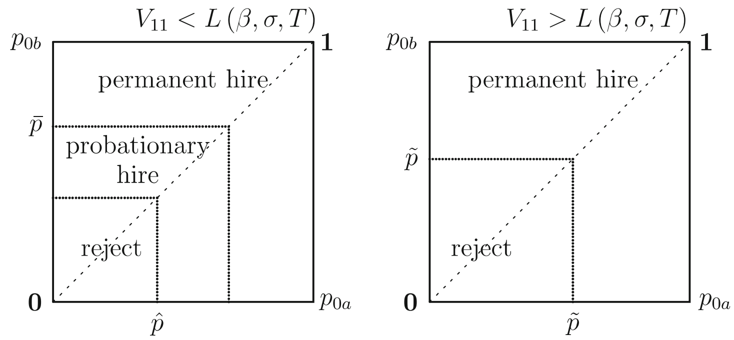

Fig. 4 Payoffs expected by member b when $(j, k) = (1, 1)$

Formally,

$$V_{11} = \tilde{p}^2 \beta V_{11} + \int_{\tilde{p}}^{1} \int_{0}^{p_{0a}} \beta^T p_{0b} \mathrm{d}p_{0b} \mathrm{d}p_{0a} + \int_{0}^{1} \int_{\max\{\tilde{p}, p_{0a}\}}^{1} \beta^T p_{0b} \mathrm{d}p_{0b} \mathrm{d}p_{0a}$$
$$= \tilde{p}^2 \beta V_{11} + \frac{1}{2} \beta^T \left(1 - \tilde{p}^3\right) \geq L(\beta, \sigma, T). \tag{13}$$

This reduces to the cubic $V_{11}^3 - 2\beta^{2T-3} V_{11} + \beta^{3T-3} = 0$. This is independent of σ: when probationary hiring is discarded, so is the value function's dependence on the quality of the signal generated during probation.

Now consider the case in which the inequalities in expression 11 hold, bounding V_{11} above by $L(\beta, \sigma, T)$. This involves all three intervals. In Fig. 4, left panel displays committee member b's expected payoffs.

Formally,

$$V_{11} = \hat{p}^2 \beta V_{11} + \int_{0}^{\hat{p}} \int_{\hat{p}}^{\bar{p}} f\left(p_{0b}\right) \mathrm{d}p_{0b} \mathrm{d}p_{0a} + \int_{\hat{p}}^{\bar{p}} \int_{0}^{\bar{p}} f\left(p_{0b}\right) \mathrm{d}p_{0b} \mathrm{d}p_{0a}$$
$$+ \int_{0}^{\bar{p}} \int_{\bar{p}}^{1} \beta^T p_{0b} \mathrm{d}p_{0b} \mathrm{d}p_{0a} + \int_{\bar{p}}^{1} \int_{0}^{1} \beta^T p_{0b} \mathrm{d}p_{0b} \mathrm{d}p_{0a}$$
$$= \hat{p}^2 \beta V_{11} + \beta^T \sigma V_{11} \left(\bar{p} + \hat{p}\right) \left(\bar{p} - \hat{p}\right)$$
$$+ \frac{1}{2} \beta^T \left\{ \left[(1 - 2\sigma) V_{11} + \sigma\right] \left(\bar{p}^3 - \hat{p}^3\right) + 1 - \bar{p}^3 \right\}$$
$$\leq L(\beta, \sigma, T). \tag{14}$$

Equations 13 and 14 are plotted in Fig. 5 by projecting V_{11} onto the (β, σ) plane for $T = 2$. In what follows, we assume that $T = 2$.[17]

[17] The results for $T > 2$ are as expected: the contours become compressed toward $\beta = 1$.

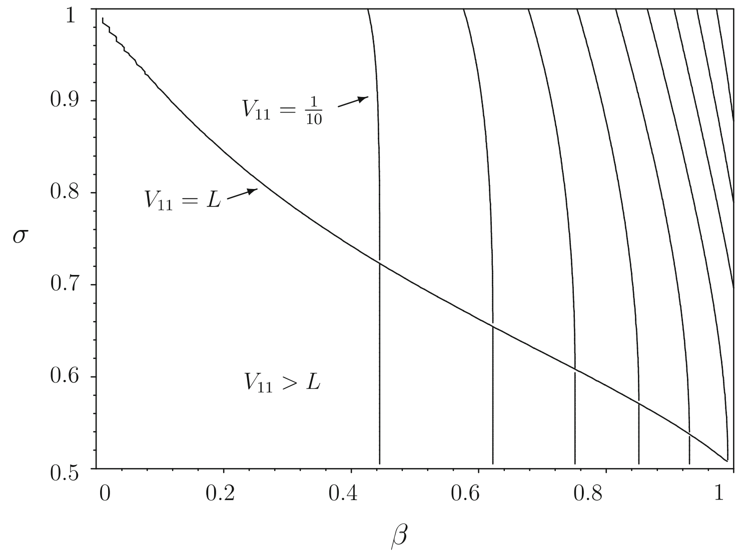

Fig. 5 Value function contours $V_{11} = \left\{\frac{1}{10}, \ldots, \frac{9}{10}\right\}$

Above the $V_{11} = L$ bisector Eq. 14 holds, while Eq. 13 holds below it, producing the region without probationary hire. As β and σ increase, the costs of probationary hire decrease: mistakes are less costly, probationary periods more informative. Thus, the region in which the committee discards probationary hire shrinks.

The other statics are also appealing. Value strictly increases in β. It increases strictly in σ when Eq. 14 holds, but is otherwise insensitive to σ, as noted above. Thus, higher values are attainable when $V_{11} < L$ than when $V_{11} > L$: informative signals are valuable.

4.2 Double unanimity: $(j, k) = (2, 2)$

Figure 6 depicts expected payoffs when $j = k = 2$ and $V_{22} > L(\beta, \sigma, T)$. While its space is divided into the same regions and by the same threshold, $\tilde{p}$, as those in Fig. 4, the boundaries are now demarcated by the less optimistic agent's beliefs. Thus,

$$V_{22} = \left[1 - (1 - \tilde{p})^2\right] \beta V_{22} + \frac{1}{2}(1 + \tilde{p})(1 - \tilde{p})^2 \beta^T \geq L(\beta, \sigma, T). \quad (15)$$

Again, this is σ independent.

When $V_{22} < L(\beta, \sigma, T)$ the expected payoffs' map resembles that in the left panel of Fig. 4, but with the boundaries again demarcated by the less optimistic agent's beliefs. Thus:

$$\begin{aligned} V_{22} = & \left[1 - \left(1 - \hat{p}\right)^2\right] \beta V_{22} + \frac{1}{2}(1 + \bar{p})(1 - \bar{p})^2 \beta^T \\ & + \beta^T \sigma V_{22} \left[2 - \left(\bar{p} + \hat{p}\right)\right] \left(\bar{p} - \hat{p}\right) + \frac{1}{2}\left[(1 - 2\sigma) V_{22} + \sigma\right] \\ & \times \beta^T \left[\left(\bar{p} + \bar{p}^2 - \bar{p}^3\right) - \left(\hat{p} + \hat{p}^2 - \hat{p}^3\right)\right] \leq L(\beta, \sigma, T). \end{aligned}$$

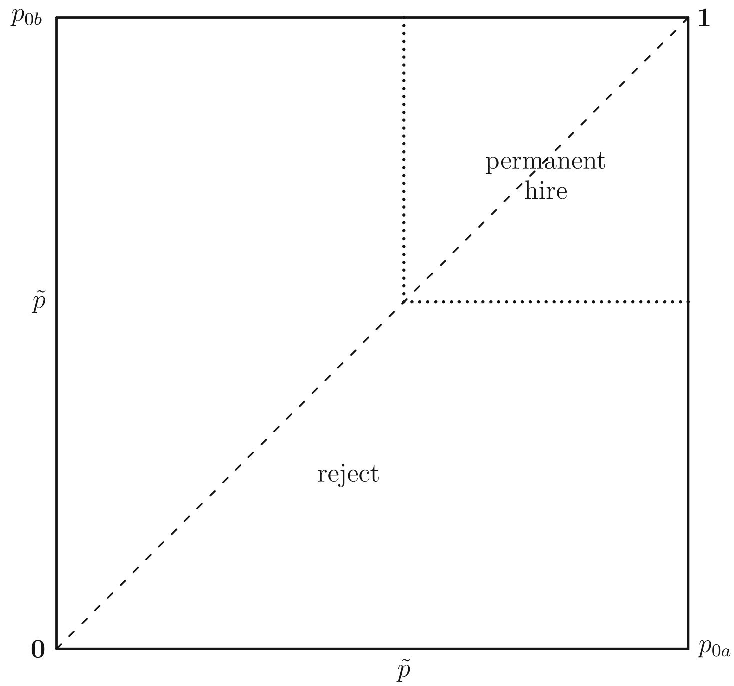

Fig. 6 Payoffs expected by member b when $(j,k)=(2,2)$, $V_{22}>L(\beta,\sigma,T)$

Its contours are qualitatively similar to those of the $j=k=1$ case.

4.3 Falling threshold: $(j,k)=(2,1)$

When $(j,k)=(2,1)$ and $V_{21}>L(\beta,\sigma,T)$, the expected payoffs' map is the same as that in Fig. 6. The value function over this range is therefore defined as in Eq. 15.

When $V_{21}<L(\beta,\sigma,T)$, expected payoffs are as displayed in Fig. 7; they are symmetric about $p_{0a}=p_{0b}$. In this case,

$$\begin{aligned} V_{21} = & \left[\hat{p}^2 + 2\left(\bar{p}-\hat{p}\right)\hat{p} + 2\left(1-\bar{p}\right)\tilde{p}\right]\beta V_{21} \\ & + \beta^T\left(\bar{p}-\hat{p}\right)^2\left\{\sigma V_{21} + \frac{1}{2}\left[(1-2\sigma)V_{21}+\sigma\right]\left(\bar{p}+\hat{p}\right)\right\} \\ & + \frac{1}{2}\beta^T\left(1-\bar{p}\right)\left[\left(1+\bar{p}\right)\left(1-\tilde{p}\right) + \left(\bar{p}^2-\tilde{p}^2\right)\right] \le L(\beta,\sigma,T). \end{aligned}$$

The contours of V_{21} are qualitatively similar to those already derived.

4.4 Rising threshold: $(j,k)=(1,2)$

When $(j,k)=(1,2)$, expected payoffs are as depicted in Fig. 8; the left panel corresponds to $V_{12}\le L(\beta,\sigma,T)$ and the right to its complement.

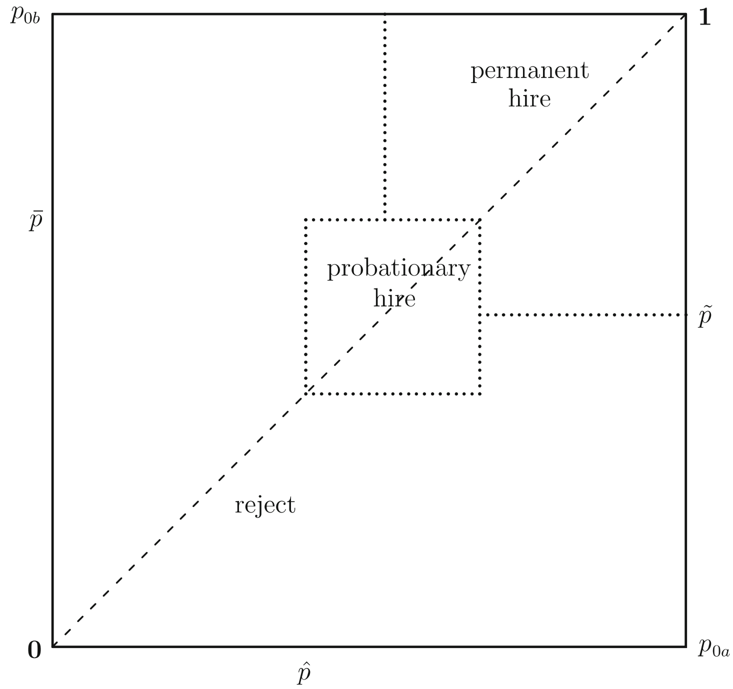

Fig. 7 Payoffs expected by member b when $(j, k) = (2, 1)$, $V_{21} < L(\beta, \sigma, T)$

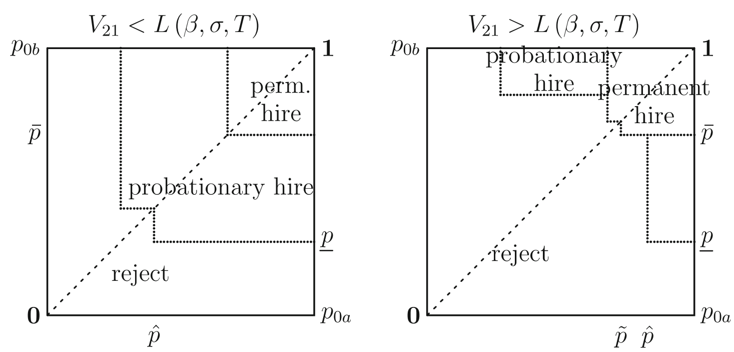

Fig. 8 Payoffs expected by member b when $(j, k) = (1, 2)$

Thus,

$$\begin{aligned} V_{12} = &\left[\tilde{p}^2 + 2\left(\hat{p} - \tilde{p}\right)\bar{p} + 2\left(1 - \hat{p}\right)\underline{p}\right]\beta V_{12} \\ &+\beta^T\left\{\frac{1}{2}\left[\left(1 - 2\sigma\right)V_{12} + \sigma\right]\left(1 - \hat{p}\right)\left(\bar{p} - \underline{p}\right)\left(1 + \hat{p} + \bar{p} + \underline{p}\right)\right. \\ &\qquad\left. +2\sigma V_{12}\left(\bar{p} - \underline{p}\right)\left(1 - \hat{p}\right)\right\} \\ &+\frac{1}{2}\beta^T\left(1 - \tilde{p}\right)\left[\left(1 + \tilde{p}\right)\left(1 - \bar{p}\right) + \tilde{p}^2 - \bar{p}^2\right] \geq L(\beta, \sigma, T); \end{aligned}$$

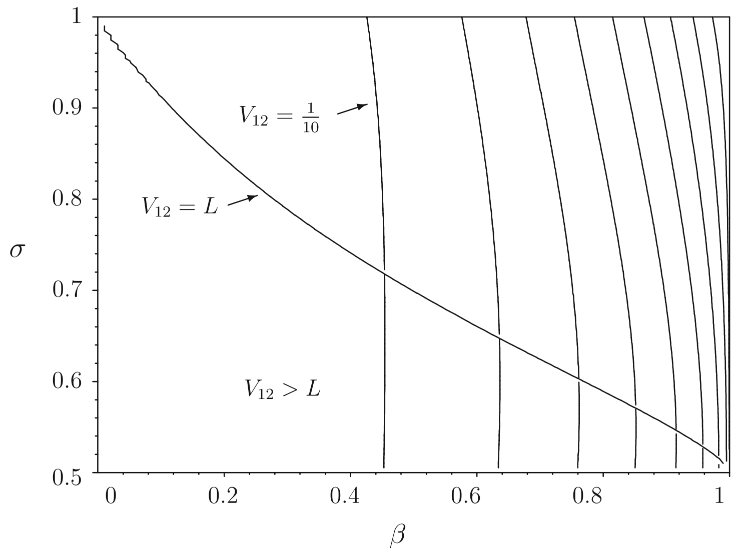

Fig. 9 Value function contours $V_{12} = \left\{\frac{1}{10}, \ldots, \frac{9}{10}\right\}$ when $j = 1, k = 2$

and

$$\begin{aligned} V_{12} = &\left[\hat{p}^2 + 2\left(1 - \hat{p}\right)\underline{p}\right]\beta V_{12} + \frac{1}{2}\beta^T\left(1 + \bar{p}\right)\left(1 - \bar{p}\right)^2 \\ &+ \frac{1}{2}\beta^T\left[(1 - 2\sigma)V_{12} + \sigma\right]\left[\bar{p}\left(1 + \bar{p} - \bar{p}^2\right) - \left(\underline{p} + \underline{p}^2\right)\right. \\ &\left.+ \hat{p}\left(\underline{p}^2 + \underline{p}\hat{p} - \hat{p}^2\right)\right] + \beta^T\sigma V_{12}\left[\left(1 - \hat{p}\right)\left(\bar{p} + \hat{p} - 2\underline{p}\right)\right. \\ &\left.+ \left(1 - \bar{p}\right)\left(\bar{p} - \hat{p}\right)\right] \le L(\beta, \sigma, T). \end{aligned}$$

This is the first case in which $V > L$ is consistent with probationary hiring, so that V depends on σ.

Figure 9 displays the contours of V_{12}. When $V_{12} \ge L(\beta, \sigma, T)$, decreased signal quality improves the problem's value to the committee. The effect is weak, with the contours' curvature near $\sigma = \frac{1}{2}$ only becoming clear under magnification. Technically, as $\sigma \to \frac{1}{2}$, $\underline{p} \to \bar{p}$, eliminating probationary hiring. Intuitively, as signal quality decreases, an optimistic hiring weather-vane ceases to hope that a good performance will convince the sceptics.

4.5 The optimal voting rule

The optimal voting rule for some (β, σ) maximises $V_{jk}(\beta, \sigma)$ by choice of j and k. Assessing this analytically requires working with polynomials of order seven and higher. In general, such polynomials are not solvable in radicals even when their coefficients are rational; thus, they are not solvable with real coefficients, as here.[18]

[18] Some quintics with rational coefficients define solvable Galois groups and are therefore solvable in radicals. However, such methods do not apply to polynomials with real coefficients, much less the large class of those generated here.

Although formulae for the roots of polynomials with rational coefficients involving multivariate hypergeometric functions exist, we do not know of generalisations of such methods to the case of polynomials with real coefficents. Thus, it is not generally possible to explicitly delineate the domain over which a particular voting rule is optimal. Instead, techniques from real algebraic geometry are first used to exactly establish the inequalities in Theorems 3 and 4, which demonstrate dominated voting rules; then, as numerical methods accurately approximate polynomials' behaviour, they are used to establish the optimal voting rule.

Lemma 5 *It must be that* $V \in (0, \beta^{T-1}]$.

Proof Establish the upper bound by noting that $V > \beta^{T-1} \Rightarrow \tilde{p} > 1$. By Lemma 2, $\tilde{p} > 1 \Rightarrow \tilde{p} > \bar{p}$. This, by inequalities 11, in turn implies that $\hat{p} \geq \tilde{p} > 1$. Thus, the committee never hires a candidate. As it is impatient, it therefore expects no return. This implies $V = 0 < \beta^{T-1}$, a contradiction.

As to the lower bound, $V = 0 \Rightarrow \bar{p} = \underline{p} = 0$, contradicting Lemma 2. □

Similarly,

Theorem 3 *For all* $(\beta, \sigma) \in [0, 1] \times \left(\frac{1}{2}, 1\right]$, $V_{22}(\beta, \sigma) \geq V_{21}(\beta, \sigma)$.

The theorem is proved in the appendix.

Theorem 4 *For all* $(\beta, \sigma) \in [0, 1] \times \left(\frac{1}{2}, 1\right]$, $V_{12}(\beta, \sigma) \geq V_{22}(\beta, \sigma)$.

The proof is again left to the appendix. It also reveals that $V_{12}(\beta, \sigma) > V_{22}(\beta, \sigma)$ over the interior, $(\beta, \sigma) \in (0, 1) \times (\frac{1}{2}, 1)$. When σ is high, so that a committee is perceptive, the more restrictive (2, 2) rule gives it fewer opportunities to use its perception: fewer candidates are hired. When a committee is imperceptive, its tenure decision largely reflects its priors; a good rule should therefore get candidates to that decision point quickly, which (2, 2) does less well than does (1, 2).

Together, Theorems 3 and 4 imply that $V_{12} \geq V_{21}$ for all (β, σ). Neither rule is more stringent than the other in the sense of requiring the candidate to garner more votes over the two periods. However, the dominated rule imposes its stricter vote at the outset, when priors have not yet been informed by probationary performance. The preferred rule allows its stricter vote to condition on this performance.

The Theorems also imply that only (1, 1) and (1, 2) need be considered as optimal rules. Explicit expressions for the (β, σ) locus setting $V_{11}(\beta, \sigma) = V_{12}(\beta, \sigma)$ may be derived by calculating the resultant of the two polynomials associated with the two voting rules, eliminating V.[19] For each of the $V \gtrless L$ cases, the corresponding Sylvester matrices are 10×10. When $V \geq L$, the relevant branch of the resultant is a degree 25 polynomial in β and σ; otherwise, it is a degree 22 polynomial. As these are largely uninterpretable, Fig. 10 proceeds numerically, plotting the performance of the optimal rules, which may be partitioned into four regions:

[19] Calculating a Groëbner basis in lexicographic order, also eliminating V, is much more computational expensive; see the appendix or Cox et al. (2007, ch.3)

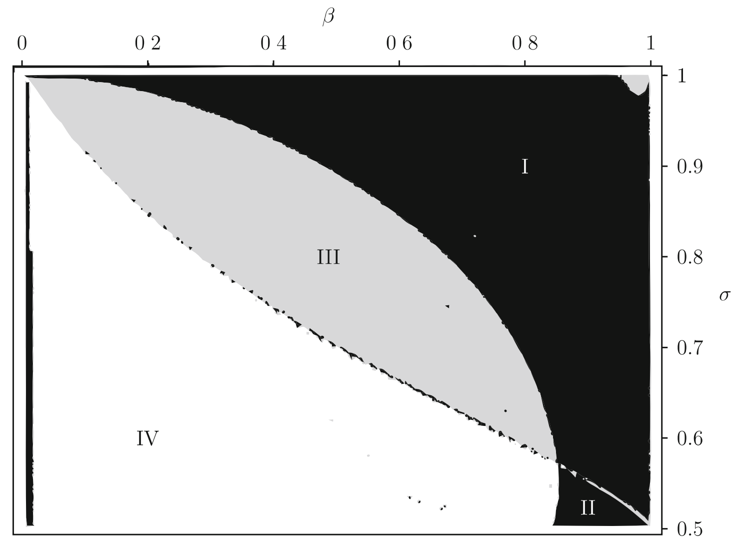

Fig. 10 Optimal voting rules

- I the rising threshold rule, (1, 2), is optimal; probationary hiring is possible (q.v. the left panel in Fig. 8): $L > V_{12} > V_{11}$.
- II the rising threshold rule is again optimal; probationary hiring is again possible (q.v. the right panel in Fig. 8), and exhibits the peculiar effect of decreased signal quality noted above: $V_{12} > V_{11}$ and $V_{12} > L$.
- III the double weak majority rule, (1, 1), is optimal; probationary hiring may occur (q.v. the left panel in Fig. 4): $L > V_{11} > V_{12}$.
- IV the double weak majority rule is again optimal, but no probationary hiring occurs (q.v. the right panel in Fig. 4): $V_{11} > V_{12}$ and $V_{11} > L$.

In regions I and II, the optimal rule allows the optimists to 'buy the option' on a candidate, but allows the sceptics to choose whether to exercise it. For a patient committee, the mistake to avoid is not a delayed hire, but a bad hire. With an informative signal, even a sceptic will be convinced by a good performance. Signal quality deterioration into region II reduces the use of probationary hiring.

In regions III and IV, the double (weak) majority rule is preferred. Scepticism about candidates when a committee is impatient and imperceptive is costly, possibly involving repeated returns to the pool. Thus, optimists are given control in both meetings, quickly hiring and tenuring. When the committee is particularly impatient or imperceptive—region IV—it even rejects probationary hiring, reducing its problem to a one-stage problem.

As a committee becomes more impatient or imperceptive, then, its optimal rule reduces the expected time to tenure a candidate: as patience or perceptiveness decrease, the majority requirement at tenure is eased, and ability to engage in probationary hiring ultimately given up.

This partition of Fig. 10 ignores two small regions: that close to $\beta = \sigma = 1$ where $V_{11} > V_{12}$ again, and that close to $\beta = 0$ for general σ. In neither case do the exact techniques of the appendix provide insight. However, these regions continue to appear even under high-resolution numerical methods, and therefore seem to represent genuine reversals.[20] However, they seem economically insignificant: at $\beta = \sigma = \frac{99}{100}$, $V_{11} \approx V_{12} + 5 \times 10^{-4}$. Thus, while not fully satisfactory to not have an explanation for this reversal, it would seem contrived to suggest one.

In the limit, analytical results become tractable, corroborating the numerical results:

Theorem 5 *In the limit as $\beta \to 0$, $V_{jk}(\beta, \sigma) \to 0$, for all voting rules. As $\beta \to 1$, $V_{12}(\beta, \sigma) \geq V_{11}(\beta, \sigma)$, reaching equality only at $\sigma = 1$. Similarly, when $\sigma = 1$, the optimal rules V_{11} and V_{12} perform equally well for all β. Finally, at $\sigma = \frac{1}{2}$, there is a unique β^0 such that $V_{11}(\beta, \sigma) > V_{12}(\beta, \sigma)$ for all $\beta \in (0, \beta^0)$, $V_{11}(\beta, \sigma) < V_{12}(\beta, \sigma)$ for all $\beta \in (\beta^0, 1)$.*

The proof is left to the appendix. The intuition for the $\beta \to 0$ result is obvious: as even the earliest tenure happens in the future under any rule, complete impatience removes all value. In the limit as committees become perfectly patient, their hiring practices become increasingly restrictive; however, under the (1, 1) rule, the beliefs of the more pessimistic committee member are ignored; as V represents the committee's expected value, this reduces V_{11} relative to V_{12} except when $\sigma = 1$, when probation is fully revealing. For any β, $\sigma = 1$ reveals the candidate's type perfectly for the tenure meeting: all committee members then vote the same way, again making the majority requirement at that stage irrelevant. Finally, when probationary signals reveal nothing, more impatient committees should impose lower barriers to tenure.

4.6 The optimal voting rule with correlated priors

Analytical results are now derived for limited cases that make weaker assumptions on the distribution of priors. Instead of assuming that (p_{0a}, p_{0b}) is drawn from the independent uniform distribution, suppose that it has density

$$g_\rho(p_{0a}, p_{0b}) = \rho\delta(p_{0a} - p_{0b}) + (1 - \rho);$$

where

$$\delta(x) \equiv \left\{ \begin{matrix} \infty \text{ if } x = 0 \\ 0 \quad \text{otherwise} \end{matrix} \right\}; \quad \text{and} \quad \int_{-\infty}^{\infty} \delta(x)\,\mathrm{d}x = 1$$

is Dirac's delta function. Thus, g_ρ is a mixture density, drawing from the UID density with probability $1 - \rho$, and from the $p_{0a} = p_{0b}$ diagonal with probability ρ. Further:

[20] Figure 10 is a generated by calculations to 50 digits over a $200 \times 200 \times 200$ grid.

Lemma 6 *Given mixture density g_ρ, the correlation of p_{0a} and p_{0b} is ρ.*

Proof As $\int_{-\infty}^{\infty} \delta(p_{0a} - p_{0b}) f(p_{0a}) \mathrm{d}p_{0a} = f(p_{0b})$, it follows that

$$E[p_{0b}] = \rho \int_0^1 \int_0^1 p_{0b}\delta(p_{0b} - p_{0a}) \mathrm{d}p_{0a}\mathrm{d}p_{0b} + \frac{1-\rho}{2} = \frac{1}{2} = E[p_{0a}];$$

$$E[p_{0b}p_{0a}] = \rho \int_0^1 \left[\int_0^1 \delta(p_{0a} - p_{0b}) p_{0a}\mathrm{d}p_{0a} \right] \mathrm{d}p_{0b} + \frac{1-\rho}{4}$$

$$= \rho \int_0^1 p_{0b}^2 \mathrm{d}p_{0b} + \frac{1-\rho}{4} = \frac{3+\rho}{12};$$

$$E\left[p_{0b}^2\right] = \rho \int_0^1 p_{0b}^2 \left[\int_0^1 \delta(p_{0a} - p_{0b}) \mathrm{d}p_{0a} \right] \mathrm{d}p_{0b} + \frac{1-\rho}{3}$$

$$= \frac{\rho}{3} + \frac{1-\rho}{3} = \frac{1}{3} = E\left[p_{0a}^2\right].$$

As the correlation coefficient is

$$\frac{E[(p_{0a} - E[p_{0a}])(p_{0b} - E[p_{0b}])]}{\sqrt{E\left[(p_{0a} - E[p_{0a}])^2\right] E\left[(p_{0b} - E[p_{0b}])^2\right]}}$$

expansion and substitution produces the result. □

Let $V_{jk}(\beta, \sigma, \rho)$ extend $V_{jk}(\beta, \sigma)$ in the natural way to a committee with correlation parameter ρ. Then:

Theorem 6 *In the limit as $\rho \to 1$, $V_{jk} - V_{j'k'} \to 0$ for all j, k, j' and k'.*

Proof In the limit, as $\rho \to 1$, all weight is placed on the uniform univariate distribution of $p_0 \equiv p_{0a} = p_{0b}$. By integrating over the relevant diagonal in Fig. 3 this produces one of two terms, depending on the inequalities in expression 11:

$$V = 1 - \sqrt{1-\beta^2} \geq L;$$

$$V = \hat{p}\beta V + \int_{\hat{p}}^{\bar{p}} \{[(1-\sigma)p_0 + \sigma(1-p_0)]V + \sigma p_0\} \beta^2 \mathrm{d}p_0 + \int_{\bar{p}}^1 \beta^2 p_0 \,\mathrm{d}p_0 \leq L.$$

□

Thus, when $\rho = 1$, agents hold the same beliefs, and vote in the same way, effectively reducing to an $N = 1$ committee. Numerical methods show that the two optimal voting rules for $N = 2$ are dominated by the $N = 1$ committee. This is a direct consequence of the committee's problem being one of social choice, not information aggregation: the single committee member is as well informed as a larger committee, but does not need to compromise. The appropriate comparison, then, is between the optimal voting rules presented above and those in which one committee member makes the decisions on behalf of both. We have shown that the (1, 1) rule is only optimal for some values of (β, σ).

This more general structure allows reconsideration of the only interior limit result from Theorem 5, namely when $\sigma = \frac{1}{2}$. While the polynomials involved are now of degree four and fifteen (up from three and twelve), they remain tractable:

Theorem 7 *Let $\beta^\rho \in (0, 1)$ solve $V^\rho \equiv V_{11}\left(\beta, \frac{1}{2}, \rho\right) = V_{12}\left(\beta, \frac{1}{2}, \rho\right)$. Then β^ρ and V^ρ are unique, and satisfy $\frac{\partial \beta^\rho}{\partial \rho}, \frac{\partial V^\rho}{\partial \rho} < 0$.*

The advantage of V_{12} relative to V_{11} is its consideration of both committee member's views; its disadvantage is that its need for broader consensus will lengthen the time taken to tenure a candidate. As committee members' views become more highly correlated, the advantage and the disadvantage of V_{12} both weaken. The theorem establishes that the advantages of taking the second member's views into account outweigh the disadvantages of lengthening the tenure process as views become more correlated.

5 Comparison to standard options model

Comparison to the discrete-time European options model in Cox et al. (1979) (CRR) clarifies both the present model's options interpretation and its differences from standard options models.

In CRR, the current value of an asset on which a call is written either increases or decreases to p^* by an exogenously determined amount. Here, priors—expected values—are exogenous; the signal quality parameter, σ, determines the extent of their updating to posteriors. CRR's time to maturity is our T. CRR's valuation formula does not depend on 'the probability that the stock price will rise or fall'; our V_{jk} is independent of $\boldsymbol{p}_0$ and the realisation of θ.

The models differ in two principal ways. First is the way in which it is closed. In CRR, the interest rate—the return to the market portfolio—is exogenous. With K, the option's strike price, the underlying asset's prices, and a no-arbitrage condition, this allows calculation of the call option's cost.

In our case, V, the expected return to the 'market', is endogenous. Endogenising this requires giving up something to avoid over-determining the system. This explains the no-arbitrage condition's absence in our model: the market for candidates is not perfectly competitive.

The second difference is that our option is managed by a committee, replacing 'rational exercise policy' by 'strategic exercise policy'. In CRR, the option's exercise depends on whether it is in the money:

$$\max\{0, p^* - K\}. \tag{16}$$

Analogously, our committee's decision to exercise the option depends on

$$\max\{0, p_{Tk} - V_{jk}\}.$$

The condition on when to buy the option is more complicated:

$$\max\left\{0, p_{0j} - \min\left\{\hat{p}, \tilde{p}\right\}\right\}. \tag{17}$$

Nevertheless, its interpretation is the same: when expression 17 is 'in the money', a candidate is hired. Which term in the min operator is smaller depends, in turn, on

$$\max\left\{0, V_{jk} - L\right\}. \tag{18}$$

As a function of exogenous parameters, but not of the voting rule, L parallels CRR's K. The upper bound of unity on L may therefore correspond to a strike price lying below an underlying asset's maximum price.

Recalling, from inequality 11, that $V_{jk} > L \Leftrightarrow \hat{p} > \tilde{p} > \bar{p}$, we may obtain an options interpretation from expression 18 as well. The committee's decision when the expression is 'in the money' depends on the voting rule. In general, it may be interpreted as an option on probationary hiring.

6 Conclusions and discussion

We have developed a model of intertemporally strategic voting when rejection of an option at either of two stages returns the committee to a candidate pool. Although voting is simultaneous at each stage, committee members do not condition on the probability of being pivotal, but on the behaviour of weather-vane committee members, generalised median voters. We identify a number of ways in which strategic considerations lead committee members to behave differently than they would as sole decision makers. As the value of rejection depends on a committee's voting rules, we derive optimal voting rules for two member committees: committees should use probationary hiring with increasing thresholds when they are patient and perceptive (in sense of receiving useful information during a probationary period) and that they avoid non-probationary hiring when they are impatient or imperceptive. We show, in limit cases, how these rules are modified as committee members' priors become correlated.

The model generates a variety of testable predictions, which seem consistent with anecdotal evidence. Formal testing would require data that do not seem to exist: the American Association of University Professors (AAUP) has a range of policy documents on tenure, none of which address voting behaviour, rules or standards. Personal correspondence with the AAUP indicates that there is not 'a comprehensive source for data on tenure decisions' or 'tenure procedural rules'; even their tenure specialists have access only to anecdotal evidence on voting behaviour and rules. Masten (2006) uses 1970 survey data on the power wielded by academics and administrators in appointment and tenure decisions. Correspondence with the (US) National Center for Education Statistics corroborates this: they 'don't have any data on the voting rules used' and 'can't think of any organization that would'. Formal empirical testing would therefore first require assembly of such a data set.

Two-stage decision processes may be seen as forms of incomplete contracts. Why do committees not, instead, write complete contracts at $t = 0$, guaranteeing tenure at $t = T$ either unconditionally or contingent upon $\theta = 1$? As our interest is in optimal voting rules, taking the two-stage committee decision as given, rather than in the optimality of that structure, we only note some possibilities in passing. A standard defence of incomplete contracts is that performance may be 'observable but non-verifiable' (q.v. Tirole 1999): academics can assess the quality of a fellow academic better than can administrators. To be rigorous, implementation theory's elicitation mechanisms must then be disabled. This, though, merely pushes the question back: why do administrators not generally financially reward or penalise academics for their votes? Answering this likely requires the addition of administrators as players to the game.[21] A second possibility that would also require a richer model is rent seeking by committee members. Third, the costs of a second meeting may be less than those of the alternatives—although modelling this is also beyond the scope of this article.

We conclude by commenting on a number of possible extensions. The first would be to derive optimal rules for committees with more than two members. Applying the present techniques present no formal problems, but are problematic for analytical results for at least two reasons: even the present analytical results are in some cases uninterpretable; some of the proof techniques used are already becoming intractable. A second extension would enrich the model's time structure: longer probationary periods could allow the committee to receive more realisations of θ (equivalently, a higher value of σ). If the committee voted in every period, such a model could be interpretable as a model of American options. A third extension would add a second signal parameter, allowing the probabilities with which a good and a bad candidates are revealed to differ. This alters the calculation of posteriors, but should retain the qualitative results as it would preserve weather-vane voters. Finally, the candidate could be made a strategic agent. Allowing him to choose an overall effort level may not have important analytical consequences if his probationary signal represented a reduced form of the effort choice; however, a candidate might seek to direct more effort at particular committee members.

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[21] Masten (2006) discusses this further, and observes that both Milgrom and Roberts (1992, pp.127-129) and Bernheim and Whinston (1998) use academic examples to motivate incomplete contracting.

Appendix: proofs

Proofs of the main results in Sects. 4.5 and 4.6 draw on techniques from algebraic geometry, which generalise linear algebra to systems of polynomials. Thus, this appendix both gives an intuitive introduction to those techniques, and then presents the proofs. More formal introductions to the techniques proving Theorems 3 and 4 are available in Cox et al. (2007) and Parrilo (2003), respectively.[22]

An *affine variety*, $\boldsymbol{V}(f_1, f_2)$ is the set of points simultaneously satisfying $f_1 = f_2 = 0$, for polynomials f_1 and f_2. Here, a voting rule, $V_{jk}(\beta, \sigma) \gtrless L$ is represented as a polynomial, $f_{jk,h}(\beta, \sigma, V)$ when $V_{jk} \geq L$ and $f_{jk,l}(\beta, \sigma, V)$ otherwise. The affine variety generated by a pair of voting rules are the set of (β, σ) such that the rules yield the same V, performing equally well. An affine variety is *irreducible* if it cannot be decomposed into simpler affine varieties. To illustrate, the variety defined by $f_1(x, y) \equiv y$ and $f_2(x, y) \equiv x^2 + y^2 - 1$ may be decomposed into the irreducible varieties, $\boldsymbol{V}(x + 1, y)$ and $\boldsymbol{V}(x - 1, y)$, corresponding to the points $(-1, 0)$ and $(1, 0)$, respectively.[23]

Varieties, as geometric objects, are closely related to *ideals*, algebraic structures, allowing translation of results between the two. Analogously to a vector space, the closure under (vector) addition and scalar multiplication of a set of basis vectors, an ideal is the closure under (polynomial) addition and polynomial multiplication of a set of basis polynomials, say $I = \langle f_1, f_2 \rangle$.

Iff $\boldsymbol{V}(f_1, f_2)$ is an irreducible variety, its corresponding ideal, $I(\boldsymbol{V}(f_1, f_2))$, is called *prime*, by analogy to prime numbers: no further factoring (or decomposition) of the polynomials will yield simpler polynomials. As an integer may make repeated use of a prime factor (e.g. $9 = 3 \times 3$), so may a prime decomposition yield multiplicity; when this multiplicity is not of interest, as here, it suffices to decompose to *radical* ideals, $\sqrt{I}$, prime ideals without the multiplicity information. Referring again to integers, a radical decomposition of 9 would just yield a single 3. Hilbert's famous *Nullstellensatz* proves a 'one-to-one correspondence between affine varieties and radical ideals' (Cox et al. 2007, p. 176): radical decomposition of an ideal therefore produces, when translated back into varieties, the union of irreducible varieties, the most tractable geometric objects. Furthermore, extensions of the Euclidean algorithm allow mechanical calculation of radical ideals. (The irreducible varieties in the illustration above are derived by radical decomposition of $I(\boldsymbol{V}(y, x^2 + y^2 - 1))$.)

Finally, if I and J are ideals, the *ideal quotient* of I by J, $I : J$, is the set of polynomials, p, such that, if q is a polynomial in J, $p \cdot q$ is in I. Thus, the ideal quotient bears resemblance to division.

Proof of Theorem 3 The case when $V_{21}, V_{22} > L$ case is trivial as it sets $V_{21} = V_{22}$.

When $V_{21}, V_{22} < L$, decomposition yields

$$\sqrt{\langle f_{21,l}, f_{22,l} \rangle} = I_1 \cap \cdots \cap I_{17};$$

[22] See Kubler and Schmedders (2007) for an application of algebraic geometry to general equilibrium settings.

[23] Irreducible varieties need not be connected: the hyperbola $x^2 + 3xy + y^2 + 2x - y + 1$, e.g. is irreducible.

Table 1 Theorem 3's decomposition of $\langle f_{21,l}, f_{22,l}\rangle$ when $V \le L$ (lex order)

Corner solutions	
I_1	$\langle \beta^3 + 2\beta^2 V^3 - 4\beta^2 V^2 - \beta^2 V - 2\beta V^3 + 7\beta V^2 - 2\beta V - V^3, \sigma - 1\rangle$
I_2	$\left\langle \beta^2 - \frac{3}{2}\beta + 1, \sigma - \frac{1}{2}, V - 1\right\rangle$
I_3	$\langle \beta^2 + 2\beta\sigma^2 - 4\beta\sigma + 1, V - 1\rangle$
I_5	$\left\langle \beta\sigma - 1, \beta V - 2V + 1, \sigma V - \frac{1}{2}\sigma - \frac{1}{2}V\right\rangle$
I_{10}	$\left\langle \beta - 1, \sigma - \frac{1}{2}, V - 1\right\rangle$
I_{11}	$\langle \beta - 1, V - 1\rangle$
I_{12}	$\left\langle \beta, \sigma V - \frac{1}{2}\sigma - \frac{1}{2}V + \frac{1}{2}\right\rangle$
I_{13}	$\langle \beta, \sigma - 1, V\rangle$
I_{14}	$\langle \beta, V\rangle$
I_{15}	$\langle \sigma - 1, V\rangle$
Empty solutions	
I_6	$\left\langle \beta + \frac{57}{26}V^2 - \frac{93}{26}V + \frac{16}{13}, \sigma - \frac{475}{26}V^2 + \frac{879}{26}V - \frac{285}{13}, V^3 - \frac{42}{19}V^2 + \frac{35}{19}V - \frac{8}{19}\right\rangle$
I_8	$\left\langle \beta + 2\sigma V^4 - 10\sigma V^3 + 11\sigma V^2 - 17\sigma V + 6\sigma - V^4 + 5V^3 - 6V^2 + 9V - 4, \sigma^2 + \frac{473}{49}\sigma V^4 - \frac{309}{7}\sigma V^3 + \frac{1921}{49}\sigma V^2 - \frac{2992}{49}\sigma V + \frac{1069}{49}\sigma - \frac{265}{49}V^4 + \frac{173}{7}V^3 - \frac{1072}{49}V^2 + \frac{1674}{49}V - \frac{613}{49}, V^5 - 5V^4 + 6V^3 - 8V^2 + 5V - 1\right\rangle$
I_9	$\langle \beta + V^4 - 5V^3 + 6V^2 - 8V + 3, \sigma - 8V^4 + 36V^3 - 30V^2 + 49V - 16, V^5 - 5V^4 + 6V^3 - 8V^2 + 5V - 1\rangle$
I_{16}	$\langle \sigma, V - 1\rangle$
I_{17}	$\langle \sigma, V\rangle$
Singleton: $(\beta, \sigma, V) \approx (0.83, 0.58, 0.36)$	
I_7	$\left\langle \beta - \frac{57}{13}V^2 + \frac{80}{13}V - \frac{32}{13}, \sigma - \frac{19}{13}V^2 + \frac{111}{52}V - \frac{15}{13}, V^3 - \frac{42}{19}V^2 + \frac{35}{19}V - \frac{8}{19}\right\rangle$
$V = L$ locus	
I_4	$\left\langle \beta^2 - 9\sigma^2 V^2 + 6\sigma^2 V - \sigma^2 + 2\sigma V^2 - 6\sigma V + 2\sigma + 2V^2 - 1, \beta\sigma - 2\sigma V + \sigma + V - 1, \beta V - \beta + 9\sigma^2 V^2 - 6\sigma^2 V + \sigma^2 - 11\sigma V^2 + 12\sigma V - 3\sigma + 3V^2 - 5V + 2, \sigma^3 V^2 - \frac{2}{3}\sigma^3 V + \frac{1}{9}\sigma^3 - \frac{11}{9}\sigma^2 V^2 + \frac{4}{3}\sigma^2 V - \frac{1}{3}\sigma^2 + \frac{5}{9}\sigma V^2 - \frac{8}{9}\sigma V + \frac{1}{3}\sigma - \frac{1}{9}V^2 + \frac{2}{9}V - \frac{1}{9}\right\rangle$

where $I_1, \ldots, I_{17}$ are prime ideals. Thus, the corresponding irreducible variety of each of these ideals is a solution to $f_{21,l} = f_{22,l} = 0$. The 17 prime ideals are listed in Table 1. Twelve describe corner solutions setting at least one of $\beta \in \{0, 1\}, \sigma \in \left\{\frac{1}{2}, 1\right\}, V \in \{0, 1\}$. Three more have no solutions in the relevant domain.

This leaves ideals I_4 and I_7. The second polynomial in I_4 is $V_{21} = V_{22} = L$, consistent with the $V_{21}, V_{22} \ge L$ case. Off of $V = L$, this ideal yields no further solutions.

Computing the ideal quotient, $I_7 : I_4$, shows, by Cox et al. (2007, Proposition 4.4.9) that $I_4 \subset I_7$. Thus, $\boldsymbol{V}(I_7) \subset \boldsymbol{V}(I_4)$ (Cox et al. 2007, Proposition 1.4.8), so that the solutions implied by I_7 are a subset of those of I_4, and need not be further considered.

As V_{22} and V_{21} are continuous, calculating test values when $V < L$ suffices to conclude that $V_{22} \ge V_{21}$ when both of them lie on the same side of L.

Finally, establish that cases in which L lies between V_{21} and V_{22} cannot occur: as V_{21} and V_{22} are continuous, and identical when $V \geq L$, both intersect L simultaneously. □

When radical decomposition does not yield tractable terms, an approach based on the *Positivstellensatz* or *Real Nullstellensatz* (Stengle 1973) may be able to show that two voting rules never perform equally well for any $(\beta, \sigma) \in [0, 1] \times (\frac{1}{2}, 1]$. The Positivstellensatz establishes that, when no common solution exists to a 'system of polynomial equations and inequalities …there exists a certain polynomial identity which bears *witness* to [this] fact' (Parrilo 2003, p.305).[24] More formally, emptiness of

$$\big\{\boldsymbol{x} \in \mathbb{R}^n : f_j(\boldsymbol{x}) \geq 0, g_k(\boldsymbol{x}) \neq 0, h_l(\boldsymbol{x}) = 0 \\ \text{for } j = 1, \ldots, s, k = 1, \ldots, t \quad \text{and} \quad l = 1, \ldots, u\big\}$$

is equivalent to the existence of an f, a g and an h, all polynomials with particular characteristics. These, are the certificates that "bear witness" to the set's emptiness. Parrilo (2003, Theorem 5.4) presents a constructive proof for finding these certificates. Attempts to actually calculate these certificates will be subject to numerical approximation error; however, it is their existence that matters. The insight exploited by Parrilo is that their derivation can be posed as a semi-definite programming problem, whose feasibility may be exactly determined.

Proof of Theorem 4 We prove the theorem in two halves, first proving it when $V \geq L$; the same technique then proves it for $V \leq L$ as well.

If

$$\Big\{(\beta, \sigma, V) \in \mathbb{R}^3 : \beta(1-\beta) \geq 0, \left(\sigma - \frac{1}{2}\right)(1-\sigma) \geq 0, V(1-V) \geq 0, \\ \beta(1-\beta) \neq 0, \left(\sigma - \frac{1}{2}\right)(1-\sigma) \neq 0, V(1-V) \neq 0, \\ f_{12,h}(\beta, \sigma, V) = 0,\ f_{22,h}(\beta, \sigma, V) = 0\Big\}$$

is empty, then $V_{12}(\beta, \sigma) > V_{22}(\beta, \sigma)$ for $V \geq L$ over $(\beta, \sigma) \in (0, 1) \times (\frac{1}{2}, 1)$. Drawing on the Positivstellensatz, the constructive approach of Parrilo (2003, Theorem 5.4) yields certificates,

$$f(\beta, \sigma, V) = p_0(\cdot) + p_1(\cdot)\beta(1-\beta) + p_2(\cdot)\left(\sigma - \frac{1}{2}\right)(1-\sigma) + p_3(\cdot)V(1-V) \\ + p_4(\cdot)\beta(1-\beta)\left(\sigma - \frac{1}{2}\right)(1-\sigma) + p_5(\cdot)\beta(1-\beta)V(1-V)$$

[24] See Sturmfels (2002, §7.4) for a full statement of the result and a discussion, including of Parrilo's approach.

$$
\begin{aligned}
&\quad +p_6(\cdot)\left(\sigma-\frac{1}{2}\right)(1-\sigma)V(1-V)\\
&\quad +p_7(\cdot)\beta(1-\beta)\left(\sigma-\frac{1}{2}\right)(1-\sigma)V(1-V);\\
g(\beta,\sigma,V) &= \beta(1-\beta)\left(\sigma-\frac{1}{2}\right)(1-\sigma)V(1-V);\ \text{and}\\
h(\beta,\sigma,V) &= q_1(\cdot)f_{12h}(\beta,\sigma,V)+q_2(\cdot)f_{22h}(\beta,\sigma,V).
\end{aligned}
$$

Furthermore, $q_1(\cdot)$, the coefficient of $f_{12,h}$ (a degree 12 polynomial) is a constant. This establishes non-intersection in the $V \geq L$ case. Evaluating $V_{12}(\beta,\sigma)$ and $V_{22}(\beta,\sigma)$ for admissible parameters establishes the inequality.

When $V \leq L$, the same approach is used. The inequality and inequity constraints remain as above, but the equality constraints are in terms of $f_{12,l}$ and $f_{22,l}$. Now, as both $f_{12,l}$ and $f_{22,l}$ are twelfth order polynomials, the problem of finding certificates is infeasible if their coefficients, q_1 and q_2, are required to be constants. When allowed to be quadratics, it becomes feasible, establishing non-intersection. Similarly, evaluating the polynomials at test points establishes the inequality.

Finally, establish that cases in which L lies between V_{12} and V_{22} cannot occur. As $V_{12} > V_{22} > L$ for small β, L is increasing in β and all three are continuous, V_{22} intersects L at the lower value of β; for higher β, $V_{12} > L > V_{22}$, consistent with the theorem; finally, $L \geq V_{12} > V_{22}$. □

Proof of Theorem 5 As $\beta \to 0$, so $\tilde{p}$, $\hat{p} \to \infty$ so, by Theorem 2 no candidate receives votes under any rule; thus, no candidate is ever elected. Further, even were a candidate tenured, it would be at a future point, and the returns fully discounted.

When $\beta = 1$, the voting rules reduce to the polynomials

$$
\begin{aligned}
0 &= \left(-20\sigma^4+40\sigma^3-29\sigma^2+9\sigma-1\right)V_{11}^5\\
&\quad +\left(64\sigma^4-128\sigma^3+92\sigma^2-28\sigma+3\right)V_{11}^4\\
&\quad +\left(-75\sigma^4+150\sigma^3-106\sigma^2+31\sigma-3\right)V_{11}^3\\
&\quad +\left(40\sigma^4-80\sigma^3+54\sigma^2-14\sigma+1\right)V_{11}^2\\
&\quad +\left(-10\sigma^4+20\sigma^3-12\sigma^2+2\sigma\right)V+\left(\sigma^4-2\sigma^3+\sigma^2\right);\\
0 &= \left(-12\sigma^4+24\sigma^3-19\sigma^2+7\sigma-1\right)V_{12}^5\\
&\quad +\left(44\sigma^4-88\sigma^3+67\sigma^2-23\sigma+3\right)V_{12}^4\\
&\quad +\left(-61\sigma^4+122\sigma^3-88\sigma^2+27\sigma-3\right)V_{12}^3\\
&\quad +\left(39\sigma^4-78\sigma^3+52\sigma^2-13\sigma+1\right)V_{12}^2\\
&\quad +\left(-11\sigma^4+22\sigma^3-13\sigma^2+2\sigma\right)V+\left(\sigma^4-2\sigma^3+\sigma^2\right).
\end{aligned}
$$

As quintics, these are not generally solvable in radicals. However, it may be seen that $V_{11}, V_{12} = 1$ is a solution to both of them. Numerical techniques also reveal a solution to V_{11} that increases in σ, reaching $V_{11} = 1$ as σ reaches 1. While $V_{12}(\beta, \sigma) = 1$ decreases continuously as β falls from 1, only this increasing function of σ yields a continuous V_{11} as β falls below 1.

When $\sigma = 1$, manipulation of $V_{11}, V_{12} \leq L$ yields, in both cases,

$$\left(5\beta - 3\beta^2 - 1\right) V^5 + \left(\beta^3 - 4\right) V^4 + \left(\beta^3 + 2\beta\right) V^3 - \beta^3 V^2 = 0.$$

Finally, at $\sigma = \frac{1}{2}$, radical decomposition yields

$$\sqrt{\langle f_{11,h}, f_{12,h}\rangle} = J_1 \cap J_2 \cap J_3; \tag{19}$$

where $J_1 \equiv \langle \beta - 1, V - 1\rangle$, $J_2 \equiv \langle \beta, V\rangle$ and $J_3 \equiv \langle \beta + \frac{1}{9}V^3 + \frac{2}{9}V^2 - \frac{5}{9}V - \frac{2}{3}, V^4 + 3V^3 - 3V^2 + 16V - 6\rangle$. Applying Descartes' rule of signs to the second polynomial in J_3 reveals a unique root of $V \in (0, 1)$, $V^0 \approx 0.39$; the first polynomial then yields a unique $\beta^0 = \frac{3V^0}{1+V^0} \approx 0.84$; this satisfies Lemma 5's requirement that $V^0 \leq \beta^0$. The first two radical ideals, J_1 and J_2, then provide equality at $\beta = 0$ and $\beta = 1$. □

Proof of Theorem 7 At $\sigma = \frac{1}{2}$, radical decomposition yields

$$\sqrt{\langle f_{11,h}, f_{12,h}\rangle} = J_1 \cap J_2 \cap J_3' \cap J_4 \cap J_5;$$

where J_1 and J_2 are defined as in the proof of Theorem 5,

$$J_3' \equiv \Big\langle \beta - \frac{1}{9}\rho V^3 + \frac{1}{9}\rho V^2 + \frac{2}{9}\rho V + \frac{1}{9}V^3 + \frac{2}{9}V^2 - \frac{5}{9}V - \frac{2}{3},$$
$$\rho V^4 - 3\rho V^2 - 2\rho V - V^4 - 3V^3 + 3V^2 - 16V + 6\Big\rangle,$$

while $J_4 \equiv \langle \beta^2 + V^2 - 2V, \rho - 1\rangle$ and $J_5 \equiv \langle \beta, \rho - 1\rangle$. These last two ideals are inapplicable for interior ρ. Thus J_3' generalises J_3, above. Manipulation of it yields

$$\frac{\partial V}{\partial \rho} = -V \frac{V^3 - 3V - 2}{4(\rho - 1)V^3 - 9V^2 + 6(1 - \rho)V - 2\rho - 16};$$
$$\frac{\partial \beta}{\partial V} = \frac{1}{9}\left[3(\rho - 1)V^2 + 2(2 - \rho)V + 5 - 2\rho\right].$$

The bounds on ρ and V make it easy to show that $\frac{\partial \beta}{\partial V} > 0$. As to $\frac{\partial V}{\partial \rho}$, its numerator is negative, so that the overall expression will have the oppositive sign of its denominator, whose single positive term is dominated by its constant. Thus, $\frac{\partial V}{\partial \rho} < 0$. Finally, $\frac{\partial \beta}{\partial \rho} = \frac{\partial \beta}{\partial V}\frac{\partial V}{\partial \rho} < 0$, and we are done. □

In the special case, J_4 and J_5 become impossible, while $\rho = 0$, J_3' reduces to J_3; thus, the decomposition above generalises that in Eq. 19. Finally, $\beta^0 \geq \beta^\rho \geq \beta^1 \approx 0.74$ and $V^0 \geq V^\rho \geq V^1 \approx 0.32$.

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