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on options & epidemics

The pricing of call and put options seems to have nothing in common with attempts to control the spread of sexually transmitted diseases. But it turns out that, in both cases, identifying and influencing the variance of a probability distribution can be more important than identifying and influencing the mean.

It is easy to see that additional volatility in the underlying asset of a call option leads to greater option value. An option holder can cash in on the added gains from an upward fluctuation but loses no more if the price fluctuates wildly downward. For example, if you own an option to buy a share of Google stock at $200, you want Google's stock price to fluctuate. In fact, option holders should be willing to trade a lower mean for a higher volatility. A person holding the Google call should prefer a world where there is an equal (1/3) chance that the price of Google stock will end up at $100, $200, or $300 than a world where there is an equal (1/3) chance that the price of Google stock will end up at $200, $210, or $220. Even though the latter distribution has a higher mean ($210 versus $200), the higher volatility of the former distribution has a bigger impact on the option value. Under the first distribution, the call option will be worth $33 (.33 x $100). The second distribution, even though it has a higher expected stock value, produces a lower call value of $10 (.33 x $10 + .33 x $20).

A folk theorem of finance theory is that whenever you identify an implicit option, there is almost always an interesting volatility story to tell. And there are implicit, or 'real,' options in all kinds of real-world settings. For example, consider an extremely stylized nuisance dispute. Imagine that Scholes and Samuelson are neighbors and that Scholes wants to stop Samuelson from singing in the morning. How should a court allocate the singing entitlement? One traditional answer (which is even codified into the Restatement (Second) of Torts § 826(a)) is that courts should give the entitlement to the litigant that the court believes to have the higher valuation. If Scholes values silence more than Samuelson values singing, then the court should give Scholes the entitlement to control whether his neighbor sings in the morning. This simple rule seems to make eminent economic sense.

But in resolving nuisance disputes, courts often go beyond merely deciding whether to enjoin singing (or pollution). Sometimes courts give the underlying entitlement to one party, but simultaneously give the other litigant a call option to buy the entitlement for a specified

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price. For example, in the famous case of *Boomer v. Atlantic Cement Co.*, 257 N.E. 2d 870 (NY 1970), a court enjoined a factory's pollution but simultaneously gave the factory the option to continue. If the factory paid their plaintiff-neighbors the court's best assessment of the monetary value of the neighbors' damages, the factory could resume polluting. In other cases, courts give the underlying nuisance entitlement to the defendant but simultaneously give the plaintiff the option of purchasing an injunction by paying the defendant a specified amount of damages. In these cases, the courts are allocating two entitlements: they are giving a call option to one side and the underlying nuisance entitlement (subject to the option) to the other.

In allocating this implicit option, courts would do well to consider the implicit volatility of litigants' valuations. From the courts' perspective, the litigant with the more speculative valuation has the higher volatility and therefore is likely to be the more efficient option holder. To see the importance of valuation volatility in a simple example, imagine that a court believes that a Resident's harm from pollution is somewhere between $5 and $105, uniformly distributed, but that the Polluter's costs of stopping pollution are somewhere between $40 and $60, uniformly distributed. Our first intuition might again be to give the initial entitlement to the Resident – because she has a higher expected value ($55 versus $50) – and the call option to the Polluter – to make up for the fact that parties may have trouble reaching agreement when the Polluter turns out to have the higher value.

But in this example, the Resident's valuation has both a higher mean and a higher variance. Because options are worth more when the underlying entitlement is more variable, it turns out that giving the Polluter the entitlement and then giving the Resident a call option produces much higher allocative efficiency. Even though from the court's perspective the Polluter has a lower expected valuation, giving it an entitlement subject to the Resident's call is more efficient because the Resident with an option to enjoin pollution for $50 will do so whenever she has a particularly high valuation. If we give the Polluter the call option instead, we can end up with a truly inefficient outcome of pollution that creates $105 of damage. When we give the Resident the call option, this never happens. This simple and admittedly stylized example shows that valuation variance can be more important than the mean in deciding legal cases. When options are at stake, we need to attend to both.

The need to attend to volatility is important whenever options come into play. A number of years ago when I was teaching at Stanford, the university had a home mortgage program. The university would lend you half the purchase price of your house, if you give the university half the appreciation at the time of the sale. The program gave the university something akin to a call option on half your house. The university didn't have to bear any cost of home depreciation, but got half the upside if the housing value increased. I had a choice of buying a house in an unincorporated (and unzoned) new section of Mountain View or a relatively staid and seasoned development just south of the campus called College Terrace. Attending to volatility, you should be able to tell which house was more subsidized.

The mathematics of epidemiology developed independently from the mathematics of option pricing. But like call option prices, the force of an epidemic also rises with both the mean and variance of an underlying distribution. The force of an STD epidemic is, like an option, a kind of 'derivative,' in that it is derived from the mean and variance of the number of partners in a population.

It is immediately intuitive that an STD is more likely to spread when the average person in a population has a larger number of sexual partners, but the *variance* in number of sexual partners in a population also positively impacts an STD's expected replication rate. Epidemiologists have modeled the force of an epidemic in populations with heterogeneous sexual frequency to equal:

$$R_0 = \rho_0 \left(\mu + \frac{\sigma^2}{\mu} \right) \text{where:}$$

ρ_0 is the product of the transmission probability per partner (sometimes referred to as the 'efficiency' of transmission) and the average duration of the disease,

μ is the mean number of partners per unit time, and

σ^2 is the variance of the number of partners.

R_0 measures the 'reproductive rate': the average number of secondary infections produced by a single index case in a population of susceptible persons. The disease rate is stable (or 'endemic') when the infector number (R_0) equals one; epidemic when greater than one; and eventually zero (the disease will die out over time) when less than one.

The equation teaches us that for any given mean, increasing the variance in the number of partners will increase the epidemiological force of a disease. The intuition for the positive impact of variance is that populations with high variances in the number of sexual partners are likely to exhibit large connected networks of sexual nodes. The few members of the population with many sexual partners are likely to form connections with one another, as well as with members of the population who have few other sexual partners. Randomly infecting someone in a high variance network is therefore likely to spread the disease quickly, through these longer connecting chains. In a population with high variance, the few people with many sexual partners are the 'superspreaders' who tend to connect the rest of the population.

The importance of variance to the epidemiological force of infection matters because human sexuality often exhibits extremely high variance in the number of sexual partners. Furthermore, as an empirical matter, the distribution of the number of sexual partners is highly skewed. The great majority of people have had only one or zero sexual partners in the last year (and only a handful during the course of their lives), but a few people report dozens or even hundreds of partners. Partnership distributions have such a heavy tail that some researchers have found evidence suggesting that human sexuality might be an example of a 'scale-free' network with an infinite variance. If human sexuality is scale-free, policies aimed at reducing heterogeneity in the number of partners are likely to be highly effective means of reducing infection.

We tend to focus on policies that reduce the mean number of sexual partners, but we should also think about the impact of policies on the variance as well. We can reduce both the mean and the variance by inducing people in the right-hand tail of the sexuality distribution to have few partners. Kathy Baker and I have suggested one indirect way of achieving this result is to promote condom use particularly for first sexual encounters (i.e., the first time two people have sex with each other). Inducing people to use condoms the first time has a dramatic impact on reducing the effective average number of partners because 46 percent of sexual pairings have sex only one time. Condoms are an effective barrier for many STDs, and hence protected first-encounter sex renders these pairings from an epidemiological standpoint a nullity. But promoting first-encounter condom use has an even larger impact on the variance because it disproportionately impacts the effective sexuality of the right-hand tail of the distribution. Baker and I found that promiscuous people are much more likely to have 'one night stands'; so, first-encounter condom use particularly mitigates the impact of superspreaders. Promoting the idea that people should use a condom in their first encounter, no matter what, is best justified as a regulation of effective sexual variance.

But policymakers can also reduce the variance by inducing people in the left-hand tail of the sexuality distribution to have *more* pairings. Indeed, the title of Steven Landsburg's recent book *More Sex is Safer Sex* builds on just this idea. More sex by the left-hand tail of the distribution can be safer sex because it reduces population variance. More pairings by relatively nonpromiscuous people can reduce the chance that an infected person will sleep with a truly promiscuous person.

Even though more sex by the left-hand tail increases the mean of distribution, it simultaneously reduces the variance and hence the force of the epidemic. Indeed, Michael Kremer earlier pointed out that reductions in the mean number of partners that simultaneously increase the variance can increase the force of an STD epidemic. The variance effect can dominate.

The real world is of course much more complicated than any single, highly stylized equation – especially one based on the assumption of random pairings. But enlightened policymakers should always ask themselves, "How does this policy impact variance?" Abstinence-only education that induces relatively nonpromiscuous people to have fewer partners can perversely increase infectivity by increasing variance.

As a nation, we lack a vocabulary for communicating with one another about volatility. Only a small fraction of the population understands what it means to say that the standard deviation of adult male height in the United States is about three inches. The particular and counterintuitive importance of variability to the value of options and the force of STD epidemics is yet another reason for teaching statistical numeracy more widely.